UNIVERSIDADE DE ÉVORA
ESCOLA DE CIÊNCIAS E TECNOLOGIA
DEPARTAMENTO DE MATEMÁTICA

Centro de Investigação em Matemática e Aplicações

Departamento de Matemática

Programa de Doutoramento em Matemática

Seminário

21 de Novembro de 2018

CLAV-Anfiteatro 1-15h

The Bounded Functional Interpretation in Proof Mining

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Abstract

Proof mining is a program that makes use of tools from mathematical logic in order to analyse mathematical proofs. This analysis is developed with the purpose of extracting quantitative information from proofs, for example in the form of effective bounds and/or algorithms. The main tools from logic are proof interpretations, introduced by Kurt Gödel in the 1950's in order to obtain consistency proofs for theories of arithmetic. The success of the proof mining program is due to the ability of extracting computational content from non-constructive proofs and to the fact that from such analysis

it is usually possible to improve the results analysed by means of weakening the hypotheses necessary to prove them, thus leading to more general results. Moreover, in the improved results the logical tools used to analyse the original proof are not visible and can therefore be read by non-logicians.

In this talk I will present the analysis of some results about a generalized version of the Halpern type Proximal Point Algorithm. Namely, a theorem by Yao and Noor [8, theorem 3.3] and a theorem by Wang and Cui [7, theorem 1] which prove, under different assumptions, the strong convergence of the algorithm to the nearest projection point onto the set of zeros of the operator. The analysis presented uses the Bounded Functional Interpretation [3] instead of the more usual Kohlenbach's Functional Monotone Interpretation [5] and comes as a natural sequel of a paper by L. Leustean and P. Pinto [6]. The analysis of the theorem by Yao-Noor provided a first example where it is possible to avoid the use of Arithmetical Comprehension. Furthermore, the elimination of the Weak Compactness argument is addressed, in analogy with [4].

This is an ongoing collaboration with L. Leustean and P. Pinto [1, 2].

Keywords: Functional interpretations, Proof Mining, Hilbert spaces, Proximal Point Algorithm.

Acknowledgements

This talk has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA), through the grant UID-MAT-04674-2013 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

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