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# Three Topics in Techniques of Finite Elements

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**Abstract**

In the scope of *Techniques of Finite Elements* (Irons [1]), several important topics are typically left outside the curricula of Mathematical Analysis of Finite Elements. These topics are hopefully relevant for a complete and balanced perspective on discretization techniques.

An Engineering approach is adopted and we are concerned with general applicability of the techniques described. Generality allows the analyst to use the same techniques and formulations to solve Maxwell's equations, Plasma flow problems, Cauchy equilibrium [2]-[3], high-speed dynamics, fracture in rocks. Any problem stated by a PDE equipped with additional

algebraic equations and ODE, including inequalities, is a candidate for discretization. Second order PDE can be directly solved by Finite Elements and higher order PDE require generalizations of the method.

Requirements intrinsic to each class of problems must be met, for example constitutive softening changes the class of the Cauchy equilibrium equations from elliptic to mixed-type (elliptic/hyperbolic) and dedicated algorithms are required to ensure mesh convergence. Problems involving convection entail the use of techniques to ensure stability of the resulting discrete system.

As much as possible, use is made of available Mathematical results and Computer Science developments. This use is increasingly relevant as the pair {problem,resolution} grows in size and realism. Theoretical foundations support generalizations, with more useful computational tools being produced on top of sound and tested algorithms.

Multiphysics is the main goal. In the Engineering world, large systems of PDE require coupled solutions for a satisfactory understanding of a given (Engineering) problem and its sensitivity.

We here chose the following three topics:

1. Finite strain constitutive modeling: hyperelasticity and elasto-plasticity. Constitutive inequalities.

2. Element technology, alternative Finite Elements and distinct discretization techniques. Constraints.

3. Linear and nonlinear solution techniques with equality constraints.

Concerning finite strain constitutive modeling, we start with classical hyperelastic models, Neo-Hookean, Ogden's and anisotropic hyperelastic models, followed by $J_2$ plasticity, general anisotropic plasticity and discuss the so-called constitutive updating algorithms [3].

For element technology, we start with classical derivations, Taig quadrilateral and related developments, mixed formulations, enhanced strains and least-squares derivations. Necessary conditions for element stability are described.

In the linear solution we focus on Frontal methods, including related clique numbering strategies and the use of multiple point constraints. Nonlinear algorithms based on Newton's method with continuation, trust region and step-size control are discussed.

The purpose here is to motivate the audience for the Engineering perspective on discretization methods, exhibit less known techniques that can illuminate the path to more adventurous and productive developments.

**Keywords:** Finite element techniques, constitutive modeling, mixed elements, nonlinear solutions, inequalities.

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