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General autonomous fishing models with Allee effects in a random environment

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Abstract

In a randomly fluctuating environment, a general fishing model uses the stochastic differential equation

$$dX(t) = f(X(t))X(t)dt + \sigma X(t)dW(t) - qE(t, X(t))X(t)dt,$$

where $X(t)$ is the size of the fished population at time t, f (of class C^1) is the arithmetic average *per capita* growth rate, $\sigma dW(t)/dt$ describes the effect of the random fluctuations on the growth rate ($W(t)$ being a standard Wiener process

and $\sigma > 0$), $E(t, X(t))$ is the fishing effort and $q > 0$ is the catchability. Here, we will consider autonomous models with $E(t, X(t)) \equiv E(X(t)) \geq 0$ of class C^1.

For the usual density dependence case (f strictly increasing and $f(+\infty)<0$), conditions for extinction and for existence of a stationary density were studied in [1]. Certain populations, however, have Allee effects (a surprising depression, accompanied by growth, of $f(x)$ for small x values) due, for example, to difficulties in finding mating partners or setting up a collective defence against predators.

In [5], under appropriate conditions, besides a particular case, the general case of f with Allee effects was studied in the absence of fishing. Here, we generalize these results to the case of fishing with general autonomous effort $E(X)$. Again, the deciding factor between population extinction and the existence of a stationary density is the signal the geometric average net (i.e. discounting fishing mortality) *per capita* growth rate takes for small population sizes.

Harvesting profit optimization and the comparison between variable effort fishing policies $E(t, X(t))$ and constant effort policies $E(t, X(t)) \equiv E$ were carried out for particular cases in [2], [3], and [4].

Keywords: Stochastic differential equations, Allee effects, random environment, general fishing models, extinction, stationary density.

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