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Relaxation results for variational nonlocal problems

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Abstract Variational problems involving nonlocal supremal functionals

$$L^\infty(\Omega;\mathbb{R}^m) \ni u \mapsto \mathrm{esssup}_{(x,y)\in\Omega\times\Omega} W(u(x),u(y)),$$

where $\Omega \subset \mathbb{R}^n$ is a bounded, open set and $W : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a suitable function, are studied. Motivated by existence theory via the direct method, it is possible to identify a necessary and sufficient condition for L^∞-weak* lower semicontinuity of these functionals, namely, separate level convexity of a symmetrized and suitably diagonalized version of the supremands. More generally, it is shown that the supremal structure of the functionals is preserved during the process of relaxation. The proof relies substantially on the connection between supremal and indicator functionals. This allows to recast the relaxation problem into characterizing weak* closures of a class of nonlocal inclusions, which is of independent interest. To illustrate the theory, explicit relaxation formulas are given for examples of functionals with different multi-well supremands. On the basis of these results it is possible to provide explicit examples to show that the relaxation of functionals

$$L^p(\Omega) \ni u \mapsto \int_\Omega \int_\Omega W(u(x),u(y))\,dx\,dy,$$

where $\Omega \subset \mathbb{R}^n$ is an open and bounded set, $1 < p < \infty$ and $W : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ a suitable integrand, is in general not of double-integral form. This proves an up to now open statement in [Pedregal, *Rev.Mat.Complut.* **29** (2016)] and [Bellido & Mora-Corral, *SIAM J. Math. Anal.* **50** (2018)].

Joint work with Carolin Kreisbeck

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