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Convergence: what's logic got to do with it?

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Abstract

Proof mining ([5]) is a program that makes use of tools from mathematical logic in order to analyse mathematical proofs. This analysis is developed with the purpose of extracting quantitative information from proofs, for example in the form of effective bounds and/or algorithms. The success of the proof mining program is due to the ability of extracting computational content from non-constructive proofs which often allows to improve the results analysed by weakening the hypotheses necessary to prove them. Moreover, in the improved results the logical tools used to analyse the original proof are not visible and can therefore be read by non-logicians. Nevertheless, the understanding of certain logical principles, as well as their strength, is crucial in order to perform the extraction of information from the proof.

In this talk I will give a soft introduction to the proof mining program focusing on the somewhat simple example of the convergence of sequences. As it turns out, this is a very fruitful example, related with Terence Tao's notion of metastability, which is has been paramount in different areas of research (cf e.g. [1, 2, 3, 4, 6, 7] and many more).

Keywords: Proof Mining, Convergence, Functional interpretations.

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