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Do I dare to divide by zero?

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Abstract. Meadows are commutative rings with a multiplicative identity element and a total multiplicative inverse operation [1], which means that it is possible to divide by zero! Two of the main classes of meadows are involutive meadows, where the inverse of zero is defined to be zero, and common meadows, that instead introduce an error term that propagates through calculations. We introduce a new algebraic structure that can be characterized axiomatically and generalizes involutive meadows, by means of an equational axiomatization, and construct some models based on the external numbers of nonstandard analysis [2] and non-archimedean fields. Again using nonstandard analysis, we also show a model for common meadows based on the real numbers and of involutive meadows based on finite fields. With similar techniques, one could also obtain meadows based on rational numbers [3]. We finish with a brief discussion on some possibilities concerning future work. (This is joint work with Emanuele Bottazzi.)

References

[1] Jan A. Bergstra and Alban Ponse. Division by Zero in Common Meadows, pages 46–61. Springer International Publishing, Cham, 2015.
[2] Bruno Dinis and Imme van den Berg. Neutrices and external numbers: a flexible number system. Monographs and Research Notes in Mathematics. CRC Press, Boca Raton, FL, 2019. With a foreword by Claude Lobry.
[3] Bruno Dinis and Emanuele Bottazzi. Flexible involutive meadows (submitted) 2023.

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