



Study Plan

School: School of Sciences and Technology
Degree: Master
Course: Mathematics and Applications (cód. 174)

Specialization * TRANSLATE ME: *****

1st Year - 1st Semester

Specialization * TRANSLATE ME: *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07600	Elements of Cryptography	Mathematics	7.5	Semester	195
MAT07601	Geometry	Mathematics	7.5	Semester	195
MAT07602	Mathematical Logic	Mathematics	7.5	Semester	195
MAT07603	Algebra	Mathematics	7.5	Semester	195

1st Year - 2nd Semester

Specialization * TRANSLATE ME: *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07604	History of Mathematics	Mathematics	7.5	Semester	195
MAT07605	Semigroups	Mathematics	7.5	Semester	195
MAT07606	Axiomatic Set Theory	Mathematics	7.5	Semester	195
MAT07607	Computational Algebra	Mathematics	7.5	Semester	195

2nd Year - 3rd Semester

Specialization * TRANSLATE ME: *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07608	Nonstandard Analysis	Mathematics	7.5	Semester	195
MAT07609	Introduction to Algebraic Geometry	Mathematics	7.5	Semester	195
MAT07610	Introduction to Algebraic Number Theory	Mathematics	7.5	Semester	195
MAT07611	Logic and Computability	Mathematics	7.5	Semester	195
Mandatory alternatives					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
	Dissertation				
	Internship				



2nd Year - 4th Semester

Specialization *** TRANSLATE ME: ***

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Mandatory alternatives					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
	Dissertation				
	Internship				

Conditions for obtaining the Degree:

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4 UC obrigatórias num total de 30 Ects { \ }newline

{ \ }newline

2^o Semestre: { \ }newline

4 UC obrigatórias num total de 30 Ects { \ }newline

{ \ }newline

3^o Semestre: { \ }newline

2 UC optativa num total de 15 Ects { \ }newline

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Para obtenção do grau, é necessário também a aprovação na Dissertação ou Relatório de Estágio, com um total de 45 ECTS, no 3.^o e 4.^o Semestre. { \ }newline

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1st Year - 1st Semester

Specialization *** TRANSLATE ME: ***

Component code	Name	Scientific Area Field	ECTS	Duration	Hours	
Group of Options						
Component code	Name	Scientific Area Field	ECTS	Duration	Hours	
MAT07612	Multi-valued Analysis and Differential Inclusions	Mathematics	7.5	Semester	194	
MAT07514	Experimental Design and Statistical Analysis	Mathematics	7.5	Semester	195	
MAT07600	Elements of Cryptography	Mathematics	7.5	Semester	195	
MAT07515	Computational Statistics	Mathematics	7.5	Semester	195	
MAT07601	Geometry	Mathematics	7.5	Semester	195	
MAT07516D	Statistical Inference	Mathematics	7.5	Semester	198	
MAT07602	Mathematical Logic	Mathematics	7.5	Semester	195	
MAT07517	Numerical Optimization	Mathematics	7.5	Semester	194	
MAT07624	Stochastic Processes	Mathematics	7.5	Semester	195	
MAT07613	Theory of Ordinary Differential Equations	Mathematics	7.5	Semester	194	
MAT07614	Topics of Functional Analysis	Mathematics	7.5	Semester	194	
MAT07603	Algebra	Mathematics	7.5	Semester	195	



1st Year - 2nd Semester

Specialization *** TRANSLATE ME: ***

Component code	Name	Scientific Area Field	ECTS	Duration	Hours	
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours	
MAT07537D	Categorical Data Analysis	Mathematics	7.5	Semester	196	
MAT07615	Numerical Analysis of Partial Differential Equations	Mathematics	7.5	Semester	194	
MAT07616	Calculus of Variations	Mathematics	7.5	Semester	194	
MAT07625	Stochastic Differential Equations and applications	Mathematics	7.5	Semester	194	
MAT07538	Multivariate Data Statistics	Mathematics	7.5	Semester	195	
MAT07604	History of Mathematics	Mathematics	7.5	Semester	195	
MAT07605	Semigroups	Mathematics	7.5	Semester	195	
MAT07539	Dynamical Systems	Mathematics	7.5	Semester	195	
MAT07540	Time Series	Mathematics	7.5	Semester	195	
MAT07606	Axiomatic Set Theory	Mathematics	7.5	Semester	195	
MAT07617	Theory of Partial Differential Equations	Mathematics	7.5	Semester	194	
MAT07607	Computational Algebra	Mathematics	7.5	Semester	195	



2nd Year - 3rd Semester

Specialization *** TRANSLATE ME: ***

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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MAT07531	Sampling Animal Populations	Mathematics	7.5	Semester	195
MAT07608	Nonstandard Analysis	Mathematics	7.5	Semester	195
MAT07534	Quality Control and Reliability	Mathematics	7.5	Semester	195
MAT07609	Introduction to Algebraic Geometry	Mathematics	7.5	Semester	195
MAT07610	Introduction to Algebraic Number Theory	Mathematics	7.5	Semester	195
MAT07535	Operations Research	Mathematics	7.5	Semester	195
MAT07611	Logic and Computability	Mathematics	7.5	Semester	195
MAT07532	Mathematical Models in Biology	Mathematics	7.5	Semester	195
MAT07618	Topics of Multi-valued Analysis and Optimization	Mathematics	7.5	Semester	194
MAT07619	Topics of Numerical Analysis	Mathematics	7.5	Semester	194
MAT07620	Topics of Ordinary Differential Equations	Mathematics	7.5	Semester	194
MAT07621	Topics of Partial Differential Equations	Mathematics	7.5	Semester	194
MAT07622	Topics of Differential Geometry	Mathematics	7.5	Semester	194
MAT07623	Topics of Dynamical Systems	Mathematics	7.5	Semester	194
Mandatory alternatives					
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2nd Year - 4th Semester

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2º Semestre: { \ } newline

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3º Semestre: { \ } newline

2 UC Optativas do Grupo III num total de 15 Ects { \ } newline

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1st Year - 1st Semester

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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07514	Experimental Design and Statistical Analysis	Mathematics	7.5	Semester	195
MAT07515	Computational Statistics	Mathematics	7.5	Semester	195
MAT07516D	Statistical Inference	Mathematics	7.5	Semester	198
MAT07624	Stochastic Processes	Mathematics	7.5	Semester	195

1st Year - 2nd Semester

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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07537D	Categorical Data Analysis	Mathematics	7.5	Semester	196
MAT07625	Stochastic Differential Equations and applications	Mathematics	7.5	Semester	194
MAT07538	Multivariate Data Statistics	Mathematics	7.5	Semester	195
MAT07540	Time Series	Mathematics	7.5	Semester	195



2nd Year - 3rd Semester

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MAT07531	Sampling Animal Populations	Mathematics	7.5	Semester	195
MAT07534	Quality Control and Reliability	Mathematics	7.5	Semester	195
MAT07535	Operations Research	Mathematics	7.5	Semester	195
MAT07532	Mathematical Models in Biology	Mathematics	7.5	Semester	195
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	Dissertation				
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2nd Year - 4th Semester

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1st Year - 1st Semester

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MAT07613	Theory of Ordinary Differential Equations	Mathematics	7.5	Semester	194
MAT07517	Numerical Optimization	Mathematics	7.5	Semester	194



1st Year - 1st Semester

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MAT07612	Multi-valued Analysis and Differential Inclusions	Mathematics	7.5	Semester	194
MAT07614	Topics of Functional Analysis	Mathematics	7.5	Semester	194

1st Year - 2nd Semester

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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07615	Numerical Analysis of Partial Differential Equations	Mathematics	7.5	Semester	194
MAT07616	Calculus of Variations	Mathematics	7.5	Semester	194
MAT07539	Dynamical Systems	Mathematics	7.5	Semester	195
MAT07617	Theory of Partial Differential Equations	Mathematics	7.5	Semester	194

2nd Year - 3rd Semester

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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT07618	Topics of Multi-valued Analysis and Optimization	Mathematics	7.5	Semester	194
MAT07619	Topics of Numerical Analysis	Mathematics	7.5	Semester	194
MAT07620	Topics of Ordinary Differential Equations	Mathematics	7.5	Semester	194
MAT07621	Topics of Partial Differential Equations	Mathematics	7.5	Semester	194
MAT07622	Topics of Differential Geometry	Mathematics	7.5	Semester	194
MAT07623	Topics of Dynamical Systems	Mathematics	7.5	Semester	194
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	Internship				

2nd Year - 4th Semester

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Conditions for obtaining the Degree:

*** TRANSLATE ME: ÁREA DE ESPECIALIZAÇÃO EM ÁLGEBRA E LÓGICA: { \ } newline

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ÁREA DE ESPECIALIZAÇÃO EM MATEMÁTICA E APLICAÇÕES: { \ } newline

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Elements of Cryptography (MAT07600)

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Geometry (MAT07601)

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Mathematical Logic (MAT07602)

1. Propositional logic

- 1.1. Syntax and Semantics. Deduction and logical implication.
- 1.2. Metatheorems of Soundness and Completeness, consistency, compatibility.
- 1.3. Fundamental properties of Propositional logic: Interpolation, compactness, decidability.

2. First-order logic

- 2.1. Syntax, deduction. Semantics, models, logical implication.
- 2.2. Metatheorems of Soundness and Completeness, compactness and the Theorem of Löwenheim-Skolem.
- 2.3. Applications: formal and informal reasoning, nonstandard models of arithmetic.

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Algebra (MAT07603)

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History of Mathematics (MAT07604)

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Semigroups (MAT07605)

- Finitely generated commutative groups.
- Finitely generated cancellative monoids.
- Numerical semigroups, minimal presentation of numerical semigroups.
- Irreducible numerical semigroups.



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Axiomatic Set Theory (MAT07606)

1. The language and axioms of Zermelo-Fraenkel and elementary consequences.
2. Well-orderings and von Neumann ordinals. Transfinite induction and recursion. Ordinal arithmetic.
3. The cumulative hierarchy.
4. Numerability, non-numerability. Cardinals and cardinal arithmetic. The continuum problem. Perfect sets. Theorem of Cantor-Bendixon.
5. Axiom of Choice and some consequences.
6. Continuum Hypothesis.

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Computational Algebra (MAT07607)

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Nonstandard Analysis (MAT07608)

Axiom of existence of non-standard numbers; infinitesimal, limited and infinitely large numbers, Leibniz's Calculus Rules. Internal and external sets, principles of permanence. External Induction. Analysis with infinitesimals, nonstandard notions of regularity of functions: S-continuity, S-derivability, S-integrability. Orders of magnitude, change of scale. One from the special topics: singular perturbations, asymptotical approximations, infinitesimal discretizations.

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Introduction to Algebraic Geometry (MAT07609)

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Introduction to Algebraic Number Theory (MAT07610)

1. The idea of integer
2. The idea of unique factorization
3. The idea of "local" and "global"
4. The idea of class field theory
5. The idea of harmonic analysis in Number Theory
6. The idea of "zeta function" in Number Theory
7. Ideas of contemporary geometry in Number Theory

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Logic and Computability (MAT07611)

Algorithms and effective computability. Turing machines. Church-Turing thesis. Primitive recursive functions. Coding, indexing, diagonalization.

Minimalization. Partial recursive functions. Recursively enumerable sets. Diophantine sets. Halting Problem and other recursively unsolvable problems. Formal Theories. Axiomatizing first order logic. Formal number theory and its models. Representability and consequences. Gödel's theorems: Incompleteness and undecidability. Theorems of Church and Tarski. Hierarchies.



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Multi-valued Analysis and Differential Inclusions (MAT07612)

Elements of Convex Analysis

Generic Derivation

Proximal Analysis

Analysis of locally lipschitzean functions

Multivalued Analysis: Differential and Integral Calculus

Differential Inclusions

Viability Theory

Applications to Optimal Control

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Experimental Design and Statistical Analysis (MAT07514)

Analysis of Variance Basic principles of experimental design. Factor concept, unrestricted randomization and block randomization, fixed and random effects, balanced and unbalanced designs. Analysis of variance assumptions. Normal test plot. Kolmogorov-Smirnov test. Levene test for equality of variances. Analysis of variance (Completely randomized experiments). The fixed effects model: single-factor experiments, randomized block design, design of experiments with several factors. The random effects model. The mixed effects model. Analysis of variance with and without replications. Multiple comparison tests: Scheffé, Tukey, Duncan, Dunnett. Orthogonal contrasts. The effects of violations of ANOVA assumptions. Nonparametric methods in the ANOVA: Kruskal-Wallis and Friedman tests. Complete and incomplete block designs. Latin squares designs. Use of statistical software (SPSS) Regression Analysis Simple linear regression and correlation. Linear correlation: Pearson's coefficient. Scatter diagram. Different types of correlation. Spearman's coefficient of correlation. Simple linear regression model. Underlying assumptions. Least squares estimation of linear regression coefficients. Properties of least squares estimators. Probability distribution of least squares estimators. Confidence intervals and hypothesis tests for regression coefficients. Model adequacy checking and validation: F test, coefficient of determination and residual analysis. Predictions from the model. Prediction intervals for future observations. Prediction intervals for the mean of future observations. Multiple linear regression. Multiple linear regression model. Underlying assumptions. Least squares estimators and their properties. Inference on linear functions of parameters. Model adequacy checking and validation. Confidence intervals and hypothesis tests for regression coefficients in multiple linear regression. Multiple coefficient of determination adjusted. Lack-of-Fit test. Residual analysis. Predictions from the model. Prediction intervals for future observations. Prediction intervals for the mean of future observations. Nonlinear regression. Simple nonlinear regression models transformed in linear regression models. Multiple nonlinear regression models transformed in multiple linear regression models. Nonlinear regression without variable transformations. Use of statistical software (SPSS) Complements of Regression Analysis Diagnostics for influential observations. Residuals and influence in regression. Outliers. Leverage. Autocorrelation Durbin-Watson test. Multicollinearity Partial and multiple correlations. Partial F tests and sequential F tests. Selection of variables in multiple linear regression. Model selection: Stepwise, Forward and backward selection. Introduction to response surfaces methodology. Analysis of Covariance. Identification of non controllable external variation sources. Single-factor analysis of covariance. Model analysis. Use of statistical software (SPSS)

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Computational Statistics (MAT07515)

p { margin-bottom: 0.08in; } Statistical modelling. Common Statistical models. Adjustment non-parametric tests. Independence tests and uniformity tests. Graphics methods. Maximum Likelihood estimation and the EM algorithm (with resource to numerical methods). Uniform pseudorandom numbers generation. Pseudorandom numbers generation with a specified distribution. Resampling methods. Monte Carlo Method. Bootstrap and Jackknife. Markov Chains Monte Carlo Methods (MCMC), Gibbs algorithm and Metropolis-Hasting algorithm. Applications and use of statistical software.



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Statistical Inference (MAT07516D)

1. Fundamental concepts of probability (measure and probability, random vectors, marginal and conditional distributions, expected values, generating and characteristics functions, functions of random vectors and transformations). 2. Review of discrete and continuous distributions properties. Exponential families. Multinormal and multinomial distributions. 3. Sampling and the most used sampling distributions. 4. Point estimation. Estimation methods (moments, maximum likelihood, least squares and bayes estimators). Properties of estimators. Crámer-Rao lower bound. Asymptotic behaviour. Robustness. 5. Interval estimation. Methods for finding interval estimators. Properties. Classical and bayesian approach. 6. Hypotheses testing. Type I and Type II probability errors. Duality. Methods for finding testes. Likelihood ratio tests. Properties of tests. Neyman-Pearson theorem, most powerful tests. Asymptotic behaviour. Robustness. Classical and bayesian approach. 7. Non-parametric tests.

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Numerical Optimization (MAT07517)

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Stochastic Processes (MAT07624)

General concepts on stochastic processes. Martingales and applications. Markov chains in discrete time. Homogeneous and non-homogeneous Poisson processes. Compound Poisson processes. Birth and death processes. Introduction to queuing theory. Renewal processes. Monte Carlo simulation methods. Some applications of the processes studied.

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Theory of Ordinary Differential Equations (MAT07613)

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Topics of Functional Analysis (MAT07614)

To choose material among the following topics: 1. Theory of distributions. Sobolev spaces. Embedding theorems. 2. Semigroups of linear operators. Hille-Yosida Theorem. Monotone operators. 3. Leray-Schauder Theory of topological degree. Nonlinear operators. Fixed points. 4. Spectral Theory of linear operators in Hilbert spaces.

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Categorical Data Analysis (MAT07537D)

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Numerical Analysis of Partial Differential Equations (MAT07615)



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Calculus of Variations (MAT07616)

1. Introduction.
2. Classic problems and indirect methods.
 - 2.1. The Euler-Lagrange differential equation and other necessary conditions for minimizers.
 - 2.2. Calibrators and sufficient conditions for existence of minimizers.
 - 2.3. Examples of classic problems of minimization of integrals.
3. The direct method for single integrals.
 - 3.1. Sobolev spaces in dimension 1.
 - 3.2. Absolutely continuous functions.
 - 3.3. Lower semicontinuity implies convexity.
 - 3.4. Convexity implies lower semicontinuity.
 - 3.5 Existence of minimizers in Sobolev spaces.
 - 3.6. Introduction to minimizers regularity theory.
 - 3.7. The DuBois-Reymond differential equation under minimal hypotheses.
 - 3.8. Linear growth integrals and positive homogeneity.
 - 3.9. Parametric integrals.
4. Vectorial integrals: Q-, P-, R-convexity.
 - 4.1. The Euler-Lagrange differential equation.
 - 4.2. Lower semicontinuity in the scalar case implies convexity.
 - 4.3. Q-, P- and R-convexity
 - 4.4. Q-convexity implies R-convexity.
 - 4.5. Lower semicontinuity implies Q-convexity.

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Stochastic Differential Equations and applications (MAT07625)

Section 1. Introduction to Stochastic Differential Equations: Wiener process and diffusion processes. Stochastic integrals. Sketched construction of the Itô integral. Use of Itô theorem. Reference to the Stratonovich integral. Existence and uniqueness theorems for stochastic differential equations (SDEs). Strong and weak solutions. Dynkin and Feynman-Kac formulas. Boundary classification for unidimensional diffusion processes. First passage times. Stationary solutions of unidimensional SDEs. Ergodicity. Monte Carlo simulation of SDEs. Section 2. Financial Applications of Stochastic Differential Equations: Black-Scholes model for stocks: detailed study, including simulation, estimation and prediction. Models for interest rates and exchange rates. Interpretation of Girsanov theorem. European and American call options and derivation of Black-Scholes formula. Cox-Ross-Rubinstein model. European put options. Generalization of the methodology to general models with several financial assets. Section 3. Complements of Stochastic Differential Equations: Detailed construction of the Itô integral. Proofs of Itô theorem, existence and uniqueness theorem for SDEs and Girsanov theorem. The Stratonovich integral, its relations with the Itô integral and their use in applications. Complements on first passage times and boundary classification of unidimensional diffusion processes. Statistical issues on SDEs, with emphasis on estimation.

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Multivariate Data Statistics (MAT07538)

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Dynamical Systems (MAT07539)



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Time Series (MAT07540)

Section 1: General Time Series Introduction to time series analysis. Introduction to stochastic processes. Linear processes: ARMA, ARIMA and SARIMA processes. Modelling linear processes Section 2: Environmental time series Application of time series linear models to environmental series. Spatial modelling. Variogram and spatial covariance. Application of spatial models to environmental real data using available software. Section 3: Economic and Business time series Application of time series linear models to economic and business time series. Transfer function models for multivariate series. Non linear processes: ARCH and GARCH processes. Modelling non linear processes. Application of nonlinear models to business and economic real data using available software.

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Theory of Partial Differential Equations (MAT07617)

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Sampling Animal Populations (MAT07531)

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Quality Control and Reliability (MAT07534)

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Operations Research (MAT07535)

Syllabus: {\}

Linear and nonlinear programming: applications, revised Simplex; interior point methods. Integer and mixed Linear programming: applications, branch and Bound methods. Nonlinear programming: applications, karushKuhnTucker conditions (KKT), evolutionary and genetic methods. {\}

Optimisation in networks, stock management and projects: Graphs: applications, settings, Matrix Representation. Trees. Flows and problems of location Algorithms for: the maximum flow problem, the shortest path and the minimum spanning tree. Project management (Pert/CPM). Stock management. {\}

Decision support systems: expected value of perfect information: decision trees. Utility Function. Multi-criteria analysis-. Game theory.

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Mathematical Models in Biology (MAT07532)

Introduction to population and ecosystem modelling. Deterministic and stochastic mathematical population growth models. Introduction to population genetic modelling. Natural resources modelling. Structured population modelling. Demographic models. Spatial dispersion models. Ecosystem modelling (competition, predation, etc.) Deterministic epidemic modelling. Population size and demographic parameters estimation.

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Topics of Multi-valued Analysis and Optimization (MAT07618)



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Topics of Numerical Analysis (MAT07619)

Representation of numbers in computing machines. Absolute and relative errors. Well and stiff conditioned problems. Differentiation, integration and interpolation. Calculation derived from any order. Quadrature formulas, adaptive methods. Associated numerical errors. Interpolation of Lagrange, Hermite and continuity C_k Splines and NURBS. Interpolation curves, surfaces and volumes. Interpolation errors. Resolution of linear and non-linear equations. Direct and iterative methods for linear systems. Methods for sparse matrices, floods and large scale systems. Newton's method, quasi-Newton for nonlinear systems. Differential equations. Approximation of functions. The finite difference method. Integration methods in time (Runge-Kutta, multistep, Newmark, ...) The finite element method. Optimization. Optimization without restrictions. Necessary conditions of extreme. Methods for functions of one variable. Methods for multiple variable functions: algorithms unused and derived from use. Optimization with constraints. Optimality conditions and Lagrange multipliers. The interior point method. Multi-objective optimization. Global optimization. Genetic algorithms. Problems of Optimal Control.

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Topics of Ordinary Differential Equations (MAT07620)

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Topics of Partial Differential Equations (MAT07621)

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Topics of Differential Geometry (MAT07622)

3 essential topics must be always available:

- Complex or Algebraic Geometry: introduction to several complex variables, complex differential operators, examples (projective varieties, symmetric spaces), sheaf theory, Hodge theory
- Riemannian Geometry: introduction, tensor calculus, representation theory of the orthogonal group, 4-manifolds, metric properties
- Symplectic Geometry: symplectic manifolds, hamiltonian field theory, Poisson structures, symplectic group actions, moment map

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Topics of Dynamical Systems (MAT07623)

Dynamical systems on graphs. Perron-Frobenius-Ruelle operator. Spectral topological invariants. Fuchsian groups, Kleinian groups. Action of discrete groups on hyperbolic space. Operator algebras associated with discrete dynamical systems. Non-linear boundary value problems. Renormalization theory on dynamical systems.