



Study Plan

School: Institute for Advanced Studies and Research
Degree: Doctorate
Course: Mathematics (E-Learning) (cód. 585)

Specialization Algebra and Logics

1st Year - 1st Semester

Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11688D	Seminary	Mathematics	12	Year	312
*** TRANSLATE ME:Grupo de Optativas ***					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11685D	Logic	Mathematics	6	Semester	156
MAT11686D	Complements of Algebra	Mathematics	6	Semester	156
MAT10149D	Combinatorial Game Theory	Mathematics	6	Semester	156
MAT11687D	Number Theory and Cryptography	Mathematics	6	Semester	156
*** TRANSLATE ME:Optativa livre ***					

1st Year - 2nd Semester

Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11689D	Geometry	Mathematics	6	Semester	156
MAT10150D	Introduction to Algebraic Geometry	Mathematics	6	Semester	156
MAT10145D	Semigroups	Mathematics	6	Semester	156
MAT10146D	Computational Algebra	Mathematics	6	Semester	156
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2nd Year - 3rd Semester

Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10147D	Complex Systems	Mathematics	6	Semester	156
MAT10148D	Nonstandard Analysis	Mathematics	6	Semester	156
Group of Free Options					



2nd Year - 3rd Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					

2nd Year - 4th Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					

3rd Year - 5th Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					

3rd Year - 6th Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					

4th Year - 7th Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					

4th Year - 8th Semester
Specialization Algebra and Logics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Specialization Analysis

1st Year - 1st Semester
Specialization Analysis

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11688D	Seminary	Mathematics	12	Year	312
*** TRANSLATE ME:Grupo de Optativas ***					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11690D	Topics in Partial Differential Equations	Mathematics	6	Semester	156
MAT11691D	Numerical Analysis of Partial Differential Equations	Mathematics	6	Semester	156
MAT11692D	Topics in Ordinary Differential Equations	Mathematics	6	Semester	156
MAT11693D	Dynamic Networks	Mathematics	6	Semester	156
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**1st Year - 2nd Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11694D	Optimization and Optimal Control	Mathematics	6	Semester	156
MAT11695D	Topics in Numerical Analysis	Mathematics	6	Semester	156
MAT11696D	Nolinear Functional Analysis and Applications	Mathematics	6	Semester	156
MAT11697D	Topics in Dynamical Systems	Mathematics	6	Semester	156
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**2nd Year - 3rd Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
*** TRANSLATE ME:Grupo de Optativas ***					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11698D	Functional Differential Equations	Mathematics	6	Semester	156
MAT11699D	Multi-valued Analysis and Differential Inclusions	Mathematics	6	Semester	156
MAT11700D	Topics of Differential Geometry and Topology	Mathematics	6	Semester	156
MAT11701D	Calculus of Variations	Mathematics	6	Semester	156
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**2nd Year - 4th Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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**3rd Year - 5th Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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**3rd Year - 6th Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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**4th Year - 7th Semester
Specialization Analysis**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					



4th Year - 8th Semester
Specialization Analysis

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Specialization Statistics

1st Year - 1st Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11688D	Seminary	Mathematics	12	Year	312
*** TRANSLATE ME:Grupo de Optativas ***					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11702D	Advanced Topics in Sampling	Mathematics	6	Semester	156
MAT11703D	Advanced Topics in Operation Research	Mathematics	6	Semester	156
MAT11704D	Advanced Topics in Stochastic Processes	Mathematics	6	Semester	156
MAT11705D	Topics in Computational Statistics	Mathematics	6	Semester	156
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1st Year - 2nd Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11706D	Advanced Topics in Experimental Delineation	Mathematics	6	Semester	156
MAT11707D	Advanced Topics in Multivariate Statistic	Mathematics	6	Semester	156
MAT11708D	Topics of Space-Time Modeling	Mathematics	6	Semester	156
MAT11709D	Tópicos in Analysis of Categorical Data	Mathematics	6	Semester	156
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2nd Year - 3rd Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11710D	Topics of Statistical Modelling	Mathematics	6	Semester	156
MAT10180D	Structural Equation Models	Mathematics	6	Semester	156
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**4th Year - 8th Semester
Specialization Statistics**

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Specialization Mathematics and Applications

**1st Year - 1st Semester
Specialization Mathematics and Applications**

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MAT11687D	Number Theory and Cryptography	Mathematics	6	Semester	156
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4th Year - 7th Semester
Specialization Mathematics and Applications

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4th Year - 8th Semester
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Conditions for obtaining the Degree:

*** TRANSLATE ME: Área de Especialização em ÁLGEBRA E LÓGICA: {\ }newline

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Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: {\ }newline

{\ }newline

1º e 2º Semestre {\ }newline

- O aluno seleccionará 60 ECTS, dos quais um mínimo de 30 ECTS de entre as UC deste 1º ano específicas deste perfil. As restantes UC's serão optativas livres de entre as restantes UC's do 1º ano do curso. {\ }newline

{\ }newline

3º Semestre {\ }newline

- O aluno seleccionará 12 ECTS de entre as UC's deste 2º ano específicas deste perfil. {\ }newline

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Para obtenção do grau, é necessário a aprovação da Tese com o total de 168 ECTS no 2º, 3º e 4º Ano {\ }newline

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Área de Especialização em ANÁLISE: {\ }newline

{\ }newline

Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: {\ }newline

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1º e 2º Semestre {\ }newline

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{\ }newline

3º Semestre {\ }newline

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Área de Especialização em ESTATÍSTICA: {\ }newline

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Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: {\ }newline

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{\ }newline

Área de Especialização em MATEMÁTICA E APLICAÇÕES: {\ }newline

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Program Contents



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Seminary (MAT11688D)

It will be invited teachers of DMAT and researchers of the CIMA-UE, preferably but not exclusively, to share their work and / or research areas.

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Logic (MAT11685D)

I. Syntax of Propositional Logic and the First-Order Logic.

II. Introduction to model theory.

2.1. Construction of the term-model.

2.2. Satisfaction, definability.

2.3. Compactness, Theorem of Löwenheim-Skolem, Skolem functions.

2.4. Homomorphisms, substructures, isomorphic models.

2.5. Elementarily equivalent models.

2.6. Saturation.

III Applications to Nonstandard Mathematics and / or O-minimal Structures.

Nonstandard Mathematics:

3.1 Ultrafilters and ultrapowers.

3.2 The elementary equivalence theorem of \aleph_1 -os.

3.3 Consistency of nonstandard axiomatics.

O-minimal Structures:

3.4 Construction of o-minimal models.

3.5 Examples of o-minimal theories.

3.5 Applications of the o-minimality in analysis, algebra and / or topology.

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Complements of Algebra (MAT11686D)

Fields and fields extensions. Simple extensions and the degree of an extension. Normability and separability. Galois theory. Applications to ruler and compass constructions. Impossibility of classical problems.

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Combinatorial Game Theory (MAT10149D)

1. Historical perspective on mathematical game theory (ies)

2. Background and mathematical notation in CGT: Conway's construction

3. Disjunctive sum: Conway's group

4. Canonical form: Domination and reversibility

5. Dyadic rationals: Simplest rule

6. Impartial games: Nimbers and the Sprague-Grundy theory

7. Hot Games: Switches and the concept of temperature

8. Infinitesimals: Atomic weight

9. The translation principle: Interpretation of the «line» of finite games



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Number Theory and Cryptography (MAT11687D)

1. Introduction of the objectives, Notion of symmetric cipher, Notion of Public-key: ciphers, signatures and protocols.
2. Number theory and cryptography. Prime divisibility, Numbers and factorization, congruences and residue class rings, function of Euler-Phi, Fermat's little theorem, and Chinese theorem of, Cyclic Groups and discrete Logarithms.
3. Symmetrical ciphers. Study of the ciphers guided to the computational efficiency, sequential ciphers for processing of information in real time, ciphers for blocks; its ways and standards: DES and AES, linear Cryptoanalysis.
4. Cryptography of public key (based in the integer factorization and the Problem of the Discrete Logarithms). The techniques RSA, Rabin and ElGamal. Digital ciphers, signatures (RSA, ElGamal and DSA), the protocol of Diffie-Hellman.
5. Cryptography based in public key in elliptic curves.

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Geometry (MAT11689D)

Elements of projective geometry. Geometrical transformations and their representation. Classification of geometries. Euclidean and non-Euclidean Geometries.

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Introduction to Algebraic Geometry (MAT10150D)

Algebraic curves and projective space. Conics and other affine curves. Singularities and tangente spaces. Rational curves, Nullstellensatz. Projective varieties.

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Semigroups (MAT10145D)

Finitely generated commutative groups.
Finitely generated cancellative monoids.
Numerical semigroups, minimal presentation of numerical semigroups.
Irreducible numerical semigroups..

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Computational Algebra (MAT10146D)

Introduction. Polynomials and affine space. Affine varieties. Parametrizations of affine varieties. Ideals. Polynomials of one variable. Monomial orderings in $k[x_1, \dots, x_n]$. A division algorithm in $k[x_1, \dots, x_n]$. Monomial ideals and Dickson's Lemma. Hilbert Basis Theorem Gröbner bases. Buchberger's algorithm. Some applications of Gröbner bases. Rewriting Systems. Knuth Bendix procedure.

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Complex Systems (MAT10147D)



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Nonstandard Analysis (MAT10148D)

1. Axioma de existência de números não-standard. Números infinitesimais, limitados e infinitamente grandes, regras de cálculo de Leibniz.
2. Conjuntos internos e externos, princípios de permanência.
3. Indução externa.
4. Análise com números infinitesimais, noções não-standard de regularidade de funções: S-continuidade, S-derivabilidade, S-integrabilidade.
5. Ordens de grandeza, mudanças de escala. Um dos tópicos especiais: perturbações singulares, aproximações assintóticas, discretizações infinitesimais.

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Topics in Partial Differential Equations (MAT11690D)

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Numerical Analysis of Partial Differential Equations (MAT11691D)

- Discretization in time and space. Finite difference method and finite element (continuous and discontinuous). Approach problems with initial and boundary conditions. Problems of Dirichlet, Neumann and Robin. Examples of applications in 2D and 3D.
- Convergence, consistency and stability.
- Parabolic equations: explicit and implicit methods using finite differences and finite elements. Application to the diffusion equation.
- Hyperbolic equations: quasi-linear and conservation formulations. Explicit and implicit methods using finite differences and finite elements.
- Elliptic equations: methods using finite differences and finite elements.
- Direct and iterative methods for solving the resulting system of equations.

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Topics in Ordinary Differential Equations (MAT11692D)

- Deformation theorems, mountain pass theorems, saddle point theorems and wedding theorem.
- Topological degree theory in finite and infinite dimensions.
- Applications to ordinary and partial differential equations.
- Fixed point theorems.
- Lower and upper solutions method applied to boundary value problems: direct method, monotone iterative method, existence of extreme solutions.

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Dynamic Networks (MAT11693D)

1. Qualitative theory of graphs
2. Understanding the basic theory of chaotic dynamical systems and stability theory
3. Relationship between the dynamic and the network structure
4. Relationship between dynamics local and global
5. Synchronization of networks
6. Spectral analysis of networks
7. Specific properties of different types of networks

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Optimization and Optimal Control (MAT11694D)



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Topics in Numerical Analysis (MAT11695D)

The course will be taught in modules. Students choose three of the following modules:

- I. Numerical modeling of fluid dynamics: Mathematical models of Newtonian fluids and non-Newtonian and analysis. Numerical methods. Applications in 1D, 2D and 3D.
- II. Numerical modeling of turbulence: Introduction to turbulence, Kolmogorov cascade, power dissipation, intermittency. Direct numerical simulation (DNS) of vortices and simulation (LES).
- III. Computational meshes: meshes uniform and nonuniform. Computational mesh generation. Adaptive meshes, structured and unstructured.
- IV. Multiprocessing: Decomposition of domains. Parallelization of algorithms. Linear algebra algorithms multiprocessor. Introduction to the implementation of finite element and finite differences for multiprocessing.
- V. Numerical methods for integral equations

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Nonlinear Functional Analysis and Applications (MAT11696D)

1. Linear Functional Differential Equations : with delay and neutral . Existence , uniqueness and continuous dependence on parameters .
- 2 . Equations in Spaces of Finite Dimension and Applications
Green operator . Problem of multipoints.
Impulsive problems of higher order
- 3 . Oscillation of Functional Differential Equations
Nonlinear differential equations with delays. Theorems Comparison and oscillation. Existence of non- oscillatory solutions .
- 4 . Impulsive Functional Problems and Stability
Lyapunov functions . Stability of solutions. Theorems on limits . Global stability relative to a parameter. Applications .
- 5 . Methods for Functional Value Problems on the Boundary. Equations with monotone operators . Iterative methods . Reduction equations . Method of lower and upper solutions
- 6 . Generalized Functional problems : adapted classical methods
Existence and multiplicity of solutions. Higher-order functional problems . Extremal solutions .

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Topics in Dynamical Systems (MAT11697D)



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Functional Differential Equations (MAT11698D)

1. Linear Functional Differential Equations

Delay-differential equations and Neutral differential equations . Generalized delay-differential equations

2. Equations in finite dimension spaces and applications

Green's functions and Green's operator.

Higher order problems (scalar case)

Multi-point problems.

Higher order impulsive problems

Equations with generalized Volterra's operator

3. Functional Differential Equations Oscillation

Comparison theorems and oscillation.

Nonlinear neutral differential equations with variable coefficients

Existence of non-oscillatory solutions

4. Functional Impulsive Problems and Stability

Stability of solutions in Lyapunov sense.

Global stability

Stability on a parameter

Applications: Population models, Neural networks, Economic models

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Multi-valued Analysis and Differential Inclusions (MAT11699D)

Elements of Convex Analysis: convex sets and functions, exposed and extreme faces, Krein-Milman theorem, duality, subdifferential, normal and tangent cones. Multifunctions in metric spaces. Continuity.

Continuous selections. Multifunctions in measurable spaces. Aumann integral. Elements of Nonsmooth Analysis: proximal analysis, Clarke's generalized gradients. Differential Inclusions. Existence theorems.

Topological and other properties of the solution set. Relaxation. Application to Optimal Control.

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Topics of Differential Geometry and Topology (MAT11700D)

Part 1 - Metric and topological spaces. Fundamental group. Covering space, universal covering space. Examples and applications.

Part 2 - Differentiable manifolds and the tangent bundle. Vector fields and orientation. Manifold-with-boundary and induced orientation. Transformations between manifolds. Brief notions of submanifold theory. Differential forms, exterior derivative. De Rham cohomology, Poincaré lemma. Integration on manifolds and Stokes theorem.

Part 3 - Riemannian manifolds and volume. Geodesics, Riemannian parallel transport. Curvature and the holonomy group. Vector bundles. Natural fibre bundles over a manifold.

Part 4 - Further topological notions, the Euler characteristic. Lie groups and álgebras. Lie group actions on manifolds. Singular homology.



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Calculus of Variations (MAT11701D)

1. Introduction.
2. Classic problems and indirect methods.
 - 2.1. The Euler-Lagrange differential equation and other necessary conditions for minimizers.
 - 2.2. Calibrators and sufficient conditions for existence of minimizers.
3. The direct method for single integrals.
 - 3.1. Sobolev spaces in dimension 1.
 - 3.2. Absolutely continuous functions.
 - 3.3. Lower semicontinuity implies convexity.
 - 3.4. Convexity implies lower semicontinuity.
 - 3.5 Existence of minimizers in Sobolev spaces.
 - 3.6. Introduction to minimizers regularity theory.
 - 3.7. The DuBois-Reymond differential equation under minimal hypotheses.
 - 3.8. Linear growth integrals and positive homogeneity.
 - 3.9. Parametric integrals.
4. Vectorial integrals: Q-, P-, R-convexity.
 - 4.1. The Euler-Lagrange differential equation.
 - 4.2. Lower semicontinuity in the scalar case implies convexity.
 - 4.3. Q-, P- and R-convexity
 - 4.4. Q-convexity implies R-convexity.
 - 4.5. Lower semicontinuity implies Q-convexity.

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Advanced Topics in Sampling (MAT11702D)

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Advanced Topics in Operation Research (MAT11703D)

1. Optimization with Genetic Algorithms{\}
2. Networks and Graphs{\}
3. Dynamic programming{\}
4. Project Management{\}
5. Productivity and Efficiency Analysis{\}
6. Markov Decision Processes

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Advanced Topics in Stochastic Processes (MAT11704D)

1. Poisson processes and its variants.
2. Renewal processes and its variants..
3. Networks of queues and applications to the modeling of telecommunications systems.
4. Diffusion process and Wiener process, Brownian motion.
5. Ito and Stratonovich stochastic integrals, Ito's formula.
6. Stochastic differential equations and its application to modeling animal population growth and financial data.



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Topics in Computational Statistics (MAT11705D)

1. Introduction to the R language.
2. Generation of Pseudo-Random Numbers.
3. Monte Carlo Methods in Statistical Inference.
4. Resampling methods: Bootstrap, Jackknife.
5. Markov Chain Monte Carlo methods (MCMC).
6. ML estimation and the EM algorithm.

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Advanced Topics in Experimental Delineation (MAT11706D)

1. Theory and practice of experimental design. Complete and balanced incomplete block designs. Latin square designs.
2. Factorial designs and fractional factorial designs.
3. Split-plot designs. Split-Block designs. Repeated measures and their relationship with the Split-Plot designs and Split Block designs.
4. Lattice Designs.
5. Crossover Designs.
6. Response Surface Methods.

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Advanced Topics in Multivariate Statistic (MAT11707D)

1. Multivariate Distributions (multivariate normal distribution, Wishart distribution, Hotelling distribution, the Wilks Lambda statistic).
2. Methods of Analysis Interdependence
3. Independent Component Analysis
4. Methods of Analysis Dependence
5. Multidimensional Scaling
6. Data Mining

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Topics of Space-Time Modeling (MAT11708D)

1. Brief review of the essential concepts of Stochastic processes.
2. Temporal linear models: SARIMA model
3. Spatial point processes
4. Continuous spatial models: kriging and co-kriging spatial interpolation methods
5. Spatial clustering analysis
6. Analysis of time series and spatial data using the software R.

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Tópicos in Analysis of Categorical Data (MAT11709D)

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Topics of Statistical Modelling (MAT11710D)

1. Quality control charts.
2. Process capability analysis and six-sigma methodology.
3. Inspection systems policies.
4. Survival models.



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Structural Equation Models (MAT10180D)

1. Introduction in Structural Equation Modeling (SEM). SEM and other multivariate techniques. Basic concepts and applications of SEM . SEM in research. Brief history of SEM.
2. Univariate and multivariate linear regression models. Exploratory Factorial Analysis. Path Analysis. Moderation effects and mediation effects with latent variables.
3. Measurement model and 1^o and 2^o order Confirmatory Factorial Analysis. Assessing measurement model validity. Construct validity.
4. Causal relationships and latent variables. Stages in testing structural theory: specification, identification, estimation, validation and modification of models. Recursive and non recursive models. Multigroup analysis. Bootstrapping in SEM.
5. Extensions. Introduction to Latent Curve Models with longitudinal data.