

#### Study Plan

School:	Institute for Research and Advanced Training
Degree:	Doctorate
Course:	Mathematics (E-Learning) (cód. 585)

#### Specialization Algebra and Logics

1st Year - 1st Semester Specialization Algebra and Logics

mponent code	Name		Scientific Area Fi	ield	EC	ΓS   D	uration	Ho
	Seminary		Mathematics		12	Ye	ar	312
AT11688D								
** TRANSLATE N	1E:Grupo de Optativas ***	·				·		
Component code	Name	Scie	entific Area Field	EC	CTS	Durati	on   I	lours
	Logic	Mat	thematics	6		Semes		56
MAT11685D								
	Complements of Algebra	Mat	thematics	6		Semest	er 1	56
MAT11686D								
	Combinatorial Game Theory	Mat	thematics	6		Semest	er 1	56
MAT10149D								
	Number Theory and Cryptography	Mat	thematics	6		Semest	er 1	56
MAT11687D								
*** TRANSLATE	ME:Optativa livre ***							

#### 1st Year - 2nd Semester

pecialization Algebra and Logics										
Component code	Name	Scientific Area Field	ECTS	Duration		Hours				
Group of Options										
Component code		Name			Scientific Area Field	ECTS	Duration	Hours		
	Geom	etry			Mathematics	6	Semester	156		
MAT11689D										
	Introd	luction to Algebraic Geor	Mathematics	6	Semester	156				
MAT10150D										
	Semig	groups			Mathematics	6	Semester	156		
MAT10145D										
	Comp	outational Algebra			Mathematics	6	Semester	156		
MAT10146D										
*** TRANSLATE	ME:Opta	ativa livre ***				•				

### 2nd Year - 3rd Semester Specialization Algebra and Logics

Specialization Algebi	ra and i	Logics									
Component code	Name	Scientific Area Field	ECTS	Duration		Hours					
Group of Options											
Component code		Name			Scientific Area Field	ECTS	Duration	Hours			
	Comp	lex Systems			Mathematics	6	Semester	156			
MAT10147D											
	Nonst	andard Analysis			Mathematics	6	Semester	156			
MAT10148D											
Group of Free Optic	ons										



# 2nd Year - 3rd Semester Specialization Algebra and Logics Component code Name Scientific Area Field ECTS Duration Hours Thesis

2nd Year - 4th Semester									
Specialization Algebra and Logics									
Component code	Name	Scientific Area Field	ECTS	Duration	Hours				
Thesis									

## 3rd Year - 5th Semester Specialization Algebra and Logics Component code Name Scientific Area Field ECTS Duration Hours Thesis

#### 3rd Year - 6th Semester

Specialization Algebra and	1 Logics
----------------------------	----------

specialization Algebra and Ebgles									
Component code	Name	Scientific Area Field	ECTS	Duration	Hours				
Thesis									

#### 4th Year - 7th Semester

Specialization	Algebra	and	Logics
Specialization	Algebia	anu	LUGICS

opecialization / lige											
Component code	Name	Scientific Area Field	ECTS	Duration	Hours						
Thesis											

 4th Year - 8th Semester

 Specialization Algebra and Logics

 Component code
 Name

 Scientific Area Field
 ECTS

 Duration
 Hours

#### **Specialization Analysis**

#### 1st Year - 1st Semester

Specialization Analysis

(	Component code	Name		Scientific Area F	Scientific Area Field ECTS Durati		d ECTS D		ion	Hours
		Seminary		Mathematics		12		Year		312
ľ	/AT11688D									
\$	*** TRANSLATE	ME:Grupo de Optativas ***								
	Component code	e Name	Sci	entific Area Field	EC	TS	Dur	ation	Ho	urs
		Topics in Partial Differential Equations	Ma	thematics	6		Sem	ester	156	
	MAT11690D									
		Numerical Analysis of Partial Differential Equations	ical Analysis of Partial Differential Equations Mathematics		6	6		lester	156	
	MAT11691D									
		Topics in Ordinary Differential Equations	Ma	thematics	6	Semester		156		
	MAT11692D									
		Dynamic Networks	Ma	thematics	6		Sem	lester	156	
	MAT11693D									
	*** TRANSLATE	E ME:Optativa livre ***								



#### **1st Year - 2nd Semester Specialization Analysis**

omponent code	Name	Scientific Area Field	ECTS	Duration		Hours		
** TRANSLATE N	/E:Grup	o de Optativas ***						
Component code		Name			Scientific Area Field	ECTS	Duration	Hours
MAT11694D	Optin	nization and Optimal Cor	ntrol		Mathematics	6	Semester	156
MAT11695D	Topic	s in Numerical Analysis			Mathematics	6	Semester	156
MAT11696D	Noline	ear Functional Analysis a	Mathematics	6	Semester	156		
MAT11697D	Topic	s in Dynamical Systems			Mathematics	6	Semester	156

#### 2nd Year - 3rd Semester Specialization Analysis

Component code	Namo	Scientific Area Field	FCTS	Duration		Hours		
Component code	Name	Scientific Area Field	ECTS	Duration		Hours		
*** TRANSLATE N	ME:Grup	o de Optativas ***						
Component code Name					Scientific Area Field	ECTS	Duration	Hours
	Funct	ional Differential Equation		Mathematics	6	Semester	156	
MAT11698D								
	Multi	-valued Analysis and Diff	erential Ir	clusions	Mathematics	6	Semester	156
MAT11699D								
	Topic	s of Differential Geometr	y and Top	ology	Mathematics	6	Semester	156
MAT11700D								
	Calcu	lus of Variations			Mathematics	6	Semester	156
MAT11701D								
*** TRANSLATE	ME:Opt	ativa livre ***			1	1		
L	-							
Thesis								

#### 2nd Year - 4th Semester

Specialization Analysis							
Component code	Name	Scientific Area Field	ECTS	Duration	Hours		
Thesis							

#### 3rd Year - 5th Semester

Specialization Analysis							
Component code	Name	Scientific Area Field	ECTS	Duration	Hours		
Thesis							

#### 3rd Year - 6th Semester Specialization Analysis

Specialization Analysis								
Component code	Name	Scientific Area Field	ECTS	Duration	Hours			
Thesis								

#### 4th Year - 7th Semester

Specialization Analysis

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis					



### 4th Year - 8th Semester Specialization Analysis Component code Name Scientific Area Field ECTS Duration Hours Thesis

#### **Specialization Statistics**

#### 1st Year - 1st Semester

Component code	Name	Scientific Area F	EC	TS   D	Duration		ours		
	Seminary		Mathematics		12	Y	ear	31	.2
/AT11688D									
*** TRANSLATE N	/E:Grupo de Optativas ***					·		·	
Component code	Name	Sci	entific Area Field	EC	TS	Durat	ion	Hours	
	Advanced Topics in Sampling	Ma	thematics	6		Semes	ter	156	
MAT11702D									
	Advanced Topics in Operation Research	Ma	Mathematics 6			Semes	ter	156	
MAT11703D									
	Advanced Topics in Stochastic Processes	Ma	Mathematics 6			Semes	ter	156	
MAT11704D									
	Topics in Computational Statistics	Ma	athematics 6			Semes	ter	156	
MAT11705D									
*** TRANSLATE	ME:Optativa livre ***								

#### **1st Year - 2nd Semester Specialization Statistics**

Component code	Name	Scientific Area Field	ECTS	Duration		Hours		
Group of Options		<u> </u>						
Component code	·   د	Name			Scientific Area Field	ECTS	Duration	Hours
MAT11706D	Advanced Topics in Experimental Delineation				Mathematics	6	Semester	156
MAT11707D	Advanced Topics in Multivariate Statistic				Mathematics	6	Semester	156
MAT11708D	Topics of Space-Time Modeling				Mathematics	6	Semester	156
MAT11709D	Tópic	s in Analysis of Categoric	al Data:		Mathematics	6	Semester	156
*** TRANSLATE	ME:Opt	ativa livre ***			1	1	1	

#### 2nd Year - 3rd Semester Specialization Statistics

Specialization Stat	131103							
Component code	Name	Scientific Area Field	ECTS	Duration		Hours		
Group of Options								
Component code	2	Name			Scientific Area Field	ECTS	Duration	Hours
	Topic	s of Statistical Modelling			Mathematics	6	Semester	156
MAT11710D								
	Struct	tural Equation Models			Mathematics	6	Semester	156
MAT10180D								
*** TRANSLATE	*** TRANSLATE ME:Optativa livre ***							



#### 2nd Year - 3rd Semester Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours		

Thesis

2nd Year - 4th Semester Specialization Statistics							
Component code	Name	Scientific Area Field	ECTS	Duration	Hours		
Thesis							

#### 3rd Year - 5th Semester

Specialization Statistics							
Component code	Name	Scientific Area Field	ECTS	Duration	Hours		
Thesis							

#### 3rd Year - 6th Semester

Specialization Statistics							
Component code	Name	Scientific Area Field	ECTS	Duration	Hours		
Thesis							

#### 4th Year - 7th Semester

Specialization Statistics								
Component code	Name	Scientific Area Field	ECTS	Duration	Hours			
Thesis								

4th Year - 8th Sen Specialization Stat	nester istics				
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Thesis			•	•	

#### **Specialization Mathematics and Applications**

#### 1st Year - 1st Semester

Specialization Mathematics and Applications

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
	Seminary	Mathematics	12	Year	312
MAT11688D					



#### 1st Year - 1st Semester Specialization Mathematics and Applications

Component code		Scientific Area Field ECTS Durat			ion	Hou			
Group of Options									
Component code	Name	Sci	Scientific Area Field		ECTS		Duration		ırs
	Logic	Ma	thematics	6		Semes	ster	156	
MAT11685D									
	Complements of Algebra	Ma	thematics	6		Semes	ster	156	
MAT11686D									
	Combinatorial Game Theory	Ma	thematics	6		Semes	ster	156	
MAT10149D									
	Number Theory and Cryptography	Ma	thematics	6		Semes	ster	156	
MAT11687D			<u> </u>	-					
MATHORD	Topics in Partial Differential Equations	Ma	thematics	6		Semes	ster	156	
MAT11690D				6		6		150	
	Numerical Analysis of Partial Differential Equations	IVIa	thematics	6		Semes	ster	150	
WAT11091D		N 4		6		<u> </u>		150	
	Topics in Ordinary Differential Equations	IVIa	thematics	0		Semes	ster	150	
WAT11092D	Dynamic Notworks	Ma	thomatics	6		Sama	tor	156	
MAT11603D	Dynamic Networks	IVIA	literitatics	0		Semes	ster	150	
WAT11095D	Advanced Tanics in Sampling	Ma	thematics	6		Seme	tor	156	
MAT11702D	Advanced Topics in Sampling	IVIA	literilatics	0		Jennes	SLEI	150	
111/020	Advanced Topics in Operation Research	Ma	thematics	6		Semes	ster	156	
MAT11703D				Ũ		Ocifics		100	
	Advanced Topics in Stochastic Processes	Ma	thematics	6		Semes	ster	156	
MAT11704D	······································			•					
	Topics in Computational Statistics	Ma	thematics	6		Semes	ster	156	
MAT11705D	, ,								
*** TRANSLATE I	ME:Optativa livre ***	1							
	•								



#### 1st Year - 2nd Semester Specialization Mathematics and Applications

Component code         Name         Scientific Area Field         ECTS         Duration         Hours								
Group of Options		•			•			
Component code		Name			Scientific Area Field	ECTS	Duration	Hours
	Geom	netry			Mathematics	6	Semester	156
MAT11689D								
	Intro	duction to Algebraic Geo	metry		Mathematics	6	Semester	156
MAT10150D	- ·					6		150
	Semi	groups			Mathematics	6	Semester	156
IVIAT 10145D	Comr	autational Algobra			Mathematics	6	Somostor	156
MAT10146D	Com	Jutational Algebra			Wathematics	0	Jennester	150
	Optin	nization and Optimal Co	ntrol		Mathematics	6	Semester	156
MAT11694D								
	Topic	s in Numerical Analysis			Mathematics	6	Semester	156
MAT11695D								
	Nolin	ear Functional Analysis a	nd Applic	ations	Mathematics	6	Semester	156
MAT11696D								
	Горіс	s in Dynamical Systems			Mathematics	6	Semester	156
IVIAT 11097D	A du a	need Tenies in Evenenings	atal Dalin		Mathematica	6	Com octor	156
MAT11706D	Auva	nceu ropics in Experimer	ital Deline	eation	Wathematics	0	Semester	150
NIX TITOOD	Adva	nced Topics in Multivaria	te Statist	ic	Mathematics	6	Semester	156
MAT11707D	, lu lu							200
	Topic	s of Space-Time Modelir	ıg		Mathematics	6	Semester	156
MAT11708D								
	Tópic	s in Analysis of Categori	cal Data		Mathematics	6	Semester	156
MAT11709D								<u> </u>
*** TRANSLATE	ME:Opt	ativa livre ***						

#### 2nd Year - 3rd Semester

#### **Specialization Mathematics and Applications**

Component code         Name         Scientific Area Field         ECTS         Duration         Hours									
0	Group of Options								
	Component code	2	Name	Scientific Area Field	ECTS	Duration	Hours		
		Comp	plex Systems	Mathematics	6	Semester	156		
	MAT10147D								
		Nons	tandard Analysis			Mathematics	6	Semester	156
	MAT10148D								
		Funct	tional Differential Equation	ons		Mathematics	6	Semester	156
	MAT11698D								
		Multi	i-valued Analysis and Diff	Mathematics	6	Semester	156		
	MAT11699D								
		Topic	cs of Differential Geometr	ology	Mathematics	6	Semester	156	
	MAT11700D								
		Calcu	ulus of Variations			Mathematics	6	Semester	156
	MAT11701D								
		Topic	cs of Statistical Modelling			Mathematics	6	Semester	156
	MAT11710D								
	*** TRANSLATE ME:Optativa livre ***								
٦	Thesis								



# 2nd Year - 4th Semester Specialization Mathematics and Applications Component code Name Scientific Area Field ECTS Duration Hours Thesis

3rd Year - 5th Semester									
Specialization Mathematics and Applications									
Component code	Name	Scientific Area Field	ECTS	Duration	Hours				
Thesis									
3rd Year - 6th Sem	lester								
Specialization Mathematics and Applications									
Component code	Name	Scientific Area Field	ECTS	Duration	Hours				
Thesis									

4th Year - 7th Semester Specialization Mathematics and Applications									
Component code	Name	Scientific Area Field	ECTS	Duration	Hours				
Thesis									

 4th Year - 8th Semester

 Specialization Mathematics and Applications

 Component code
 Name
 Scientific Area Field
 ECTS
 Duration
 Hours

 Thesis



#### Conditions for obtaining the Degree:

\*\*\* TRANSLATE ME: Área de Especialização em ÁLGEBRA E LÓGICA:{\}newline

 $\{ \setminus \}$  newline Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: { \ } newline  $\{ \setminus \}$  newline 1<sup>**e**</sup> e 2<sup>**e**</sup> Semestre  $\{ \setminus \}$  newline - O aluno seleccionará 60 ECTS, dos quais um mínimo de 30 ECTS de entre as UC deste 1º ano especificas deste perfil. As restantes UC's serão optativas livres de entre as restantes UC's do 1º ano do curso. {  $\setminus$  } newline  $\{ \}$  newline 3<sup>**2**</sup> Semestre  $\{ \setminus \}$  newline - O aluno seleccionará 12 ECTS de entre as UC's deste 2ª ano especificas deste perfil. {  $\backslash$  } newline  $\{ \setminus \}$  newline Para obtenção do grau, é necessário a aprovação da Tese com o total de 168 ECTS no 2°, 3° e 4° Ano{ $}$  pewline  $\{ \}$  newline Área de Especialização em ANÁLISE: { \ } newline  $\{ \setminus \}$  newline Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: {\} newline  $\{ \setminus \}$  newline 1º e 2º Semestre {  $\setminus$  } newline - O aluno seleccionará 60 ECTS, dos quais um mínimo de 30 ECTS de entre as UC deste 1º ano especificas deste perfil. As restantes UC's serão optativas livres de entre as restantes UC's do 1º ano do curso. {  $\setminus$  } newline  $\{ \}$  newline  $3^{2}$  Semestre { \ } newline - O aluno seleccionará 12 ECTS de entre as UC's deste 2º ano especificas deste perfil. {  $\$  } newline  $\{ \setminus \}$  newline Para obtenção do grau, é necessário a aprovação da Tese com o total de 168 ECTS no 2º, 3º e 4º Ano{\}newline  $\{ \setminus \}$  newline Área de Especialização em ESTATÍSTICA: {  $\setminus$  } newline  $\{ \setminus \}$  newline Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: { \ } newline  $\{ \setminus \}$  newline 1<sup>**e**</sup> e 2<sup>**e**</sup> Semestre {  $\setminus$  } newline - O aluno seleccionará 60 ECTS, dos quais um mínimo de 30 ECTS de entre as UC deste 1º ano especificas deste perfil. As restantes UC's serão optativas livres de entre as restantes UC's do 1º ano do curso.  $\{ \setminus \}$  newline  $\{\, \setminus\,\}\, \mathsf{newline}$ 3° Semestre {  $\setminus \ }$  newline - O aluno seleccionará 12 ECTS de entre as UC's deste 2º ano especificas deste perfil. { \ } newline  $\{ \setminus \}$  newline  $\{ \setminus \}$  newline Área de Especialização em MATEMÁTICA E APLICAÇÕES: { \ } newline { \ } newline Para aprovação na componente curricular nesta especialização deste programa de doutoramento é necessário a aprovação (através de avaliação ou creditação) das seguintes unidades curriculares: {\} newline  $\{ \setminus \}$  newline  $1^{\mathbf{e}} \in 2^{\mathbf{e}}$  Semestre { \ } newline - O aluno seleccionará 60 ECTS de entre as UC's deste 1º ano especificas deste perfil. {  $\$  } newline  $\{ \setminus \}$  newline  $3^{2}$  Semestre { \ } newline - O aluno seleccionará 12 ECTS de entre as UC's deste 2º ano especificas deste perfil. { \ } newline  $\{ \setminus \}$  newline Para obtenção do grau, é necessário a aprovação da Tese com o total de 168 ECTS no 2º, 3º e 4º Ano \*\*\*

#### **Program Contents**



#### Seminary (MAT11688D)

It will be invited teachers of DMAT and researchers of the CIMA-UE, preferably but not exclusively, to share their work and / or research areas.

#### Back

#### Logic (MAT11685D)

I. Syntax of Propositional Logic and the First-Order Logic.

- II. Introduction to model theory.
- 2.1. Construction of the term-model.
- 2.2. Satisfaction, definability.
- 2.3. Compactness, Theorem of Löwenheim-Skolem, Skolem functions.
- 2.4. Homomorphisms, substructures, isomorphic models.
- 2.5. Elementarily equivalent models.
- 2.6. Saturation.

III Applications to Nonstandard Mathematics and / or O-minimal Structures.

- Nonstandard Mathematics:
- 3.1 Ultrafilters and ultrapowers.
- 3.2 The elementary equivalence theorem of &#321;os.
- 3.3 Consistency of nonstandard axiomatics.
- O-minimal Structures:
- 3.4 Construction of o-minimal models.
- 3.5 Examples of o-minimal theories.
- 3.5 Applications of the o-minimality in analysis, algebra and / or topology.

#### Back

#### Complements of Algebra (MAT11686D)

Fields and fields extensions. Simple extensions and the degree of an extension. Normability and separability. Galois theory. Applications to ruler and compass constructions. Impossibility of classical problems.

#### Back

#### Combinatorial Game Theory (MAT10149D)

- 1. Historical perspective on mathematical game theory (ies)
- 2. Background and mathematical notation in CGT: Conway?s construction
- 3. Disjunctive sum: Conway's group
- 4. Canonical form: Domination and reversibility
- 5. Dyadic rationals: Simplest rule
- 6. Impartial games: Nimbers and the Sprague-Grundy theory
- 7. Hot Games: Switches and the concept of temperature
- 8. Infinitesimals: Atomic weight
- 9. The translation principle: Interpretation of the «line» of finite games



#### Number Theory and Cryptography (MAT11687D)

1.Introduction of the objectives, Notion of symmetricl cipher, Notion of Public-key: ciphers, signatures and protocols.

2. Number theory and cryptography. Prime divisibility, Numbers and factorization, congruences and residue class rings, function of Euler-Phi, Fermat's little theorem, and Chinese theorem of, Cyclic Groups and discrete Logarithms.

3.Symmetrical ciphers. Study of the ciphers guided to the computational efficiency, sequential ciphers for processing of information in real time, ciphers for blocks; its ways and standards: DES and AES, linear Cryptoanalysis.

4. Cryptography of public key (based in the integer factorization and the Problem of the Discrete Logarithms). The techniques RSA, Rabin and ElGamal. Digital ciphers, signatures (RSA, ElGamal and DSA), the protocol of Diffie-Hellman.

5. Cryptography based in public key in elliptic curves.

#### Back

#### Geometry (MAT11689D)

Elements of projective geometry. Geometrical transformations and their representation. Classification of geometries. Euclidean and non-Euclidean Geometries.

#### Back

#### Introduction to Algebraic Geometry (MAT10150D)

Algebraic curves and projective space. Conics and other afine curves. Singularities and tangente spaces. Rational curves, Nullstellensatz. Projective varieties.

#### Back

#### Semigroups (MAT10145D)

Finitely generated commutative groups. Finitely generated cancellative monoids. Numerical semigroups, minimal presentation of numerical semigroups. Irreducible numerical semigroups..

#### Back

#### Computational Algebra (MAT10146D)

Introduction. Polynomials and affine space. Affine varieties. Parametrizations of affine varieties. Ideals. Polynomials of one variable. Monomial orderings in kx\_1,...,x\_n. A division algorithm in kx\_1,...,x\_n. Monomial ideals and Dickson's Lemma. Hilbert Beses Theorem Gröbner bases. Buchberger's algorithm. Some applicatuons of Gröbner bases. Rewriting Systems. Knuth Bendix procedure.

#### Back

Complex Systems (MAT10147D)



#### Nonstandard Analysis (MAT10148D)

1. Axioma de existência de números não-standard. Números infinitesimais, limitados e infinitamente

grandes, regras de cálculo de Leibniz.

2. Conjuntos internos e externos, princípios de permanência.

3. Indução externa.

4. Análise com números infinitesimais, noções não-standard de regularidade de funções: S-continuidade,

S-derivabilidade, S-integrabilidade.

5. Ordens de grandeza, mudanças de escala. Um dos tópicos especiais: perturbações singulares,

aproximações assimptóticas, discretizações infinitesimais.

#### Back

#### Topics in Partial Differential Equations (MAT11690D)

#### Back

#### Numerical Analysis of Partial Differential Equations (MAT11691D)

Discretization in time and space. Finite difference method and finite element (continuous and discontinuous).

Approach problems with initial and boundary conditions. Problems of Dirichlet, Neumann and Robin. Examples of applications in 2D and 3D.

Convergence, consistency and stability.

Parabolic equations: explicit and implicit methods using finite differences and finite elements. Application to the diffusion equation.

Hyperbolic equations: quasi-linear and conservation formulations. Explicit and implicit methods using finite differences and finite elements.

Elliptic equations: methods using finite differences and finite elements.

Direct and iterative methods for solving the resulting system of equations.

#### Back

#### Topics in Ordinary Differential Equations (MAT11692D)

Deformation theorems, mountain pass theorems, saddle point theorems and wedding theorem.

Topological degree theory in finite and infinite dimensions.

Applications to ordinary and partial differential equations.

Fixed point theorems.

Lower and upper solutions method applied to boundary value problems: direct method, monotone iterative method , existence of extreme solutions.

#### Back

#### Dynamic Networks (MAT11693D)

1. Qualitative theory of graphs

- 2. Understanding the basic theory of chaotic dynamical systems and stability theory
- 3. Relationship between the dynamic and the network structure
- 4. Relationship between dynamics local and global
- 5. Synchronization of networks
- 6. Spectral analysis of networks
- 7. Specific properties of different types of networks

#### Back

#### **Optimization and Optimal Control (MAT11694D)**



#### Topics in Numerical Analysis (MAT11695D)

The course will consist in two parts. The first part aims to give the theoretical bases for numerical modelling of problems in various areas (engineering, physics, medicine, etc.) and treat specific cases through "problems models". Two methods will be covered: 1) discretization using finite differences.

2) using a variational formulation discretization. Will be treated the notion of weak solution.

The second part consists of an option to choose between: "finite difference method for problems in 2 dimensions" or "evolution problem" with explicit and implicit formulations.

In addition to the study of matter, it will be asked the students to perform research or demonstration of results.

#### Back

#### Nolinear Functional Analysis and Applications (MAT11696D)

1. Linear Functional Differential Equations : with delay and neutral . Existence , uniqueness and continuous dependence on parameters .

2. Equations in Spaces of Finite Dimension and Applications

Green operator . Problem of multipoints.

Impulsive problems of higher order

3. Oscillation of Functional Differential Equations

Nonlinear differential equations with delays. Teoremas Comparison and oscillation. Existence of non- oscillatory solutions .

4. Impulsive Functional Problems and Stability

Lyapunov functions . Stability of solutions. Theorems on limits . Global stability relative to a parameter. Applications .

5. Methods for Functional Value Problems on the Boundary. Equations with monotone operators . Iterative methods . Reduction equations . Method of lower and upper solutions

6 . Generalized Functional problems : adapted classical methods

Existence and multiplicity of solutions. Higher-order functional problems . Extremal solutions .

#### Back

#### Topics in Dynamical Systems (MAT11697D)

#### Back

#### Functional Differential Equations (MAT11698D)

Linear Functional Differential Equations
 Delay-differential equations and Neutral differential equations. Generalized delay-differential equations
 Equations in finite dimension spaces and applications
 Green 's functions and Green 's operator.

Higher order problems (scalar case)

Multi-point problems.

Higher order impulsive problems

Equations with generalized Volterra's operator

3. Functional Differential Equations Oscillation

Comparison theorems and oscillation.

Nonlinear neutral differential equations with variable coefficients

Existence of non-oscillatory solutions

4. Functional Impulsive Problems and Stability

Stability of solutions in Lyapunov sense.

Global stability

Stability on a parameter

Applications: Population models, Neural networks, Economic models



#### $\mathsf{Back}$

#### Multi-valued Analysis and Differential Inclusions (MAT11699D)

Elements of Convex Analysis: convex sets and functions, exposed and extreme faces, Krein-Milman theorem, duality, subdifferential, normal and tangent cones. Multifunctions in metric spaces. Continuity. Continuous selections. Multifunctions in measurable spaces. Aumann integral. Elements of Nonsmooth Analysis: proximal analysis, Clarke's generalized gradients. Differential Inclusions. Existence theorems. Topological and other properties of the solution set. Relaxation. Application to Optimal Control.

#### Back

#### Topics of Differential Geometry and Topology (MAT11700D)

Part 1 - Metric and topological spaces. Fundamental group. Covering space, universal covering space. Examples and applications. Part 2 - Differentiable manifolds and the tangent bundle. Vector fields and orientation. Manifold-with-boundary and induced orientation. Transformations between manifolds. Brief notions of submanifold theory. Differential forms, exterior derivative. De Rham cohomology, Poincaré lemma. Integration on manifolds and Stokes theorem.

Part 3 - Riemannian manifolds and volume. Geodesics, Riemannian parallel transport. Curvature and the holonomy group. Vector bundles. Natural fibre bundles over a manifold.

Part 4 - Further topological notions, the Euler characteristic. Lie groups and álgebras. Lie group actions on manifolds. Singular homology.

#### Back

#### Calculus of Variations (MAT11701D)

1. Introduction.

- 2. Classic problems and indirect methods.
- 2.1. The Euler-Lagrange differential equation and other necessary conditions for minimizers.
- 2.2. Calibrators and sufficient conditions for existence of minimizers.
- 3. The direct method for single integrals.
- 3.1. Sobolev spaces in dimension 1.
- 3.2. Absolutely continuous functions.
- 3.3. Lower semicontinuity implies convexity.
- 3.4. Convexity implies lower semicontinuity.
- 3.5 Existence of minimizers in Sobolev spaces.
- 3.6. Introduction to minimizers regularity theory.
- 3.7. The DuBois-Reymond differential equation under minimal hypotheses.
- 3.8. Linear growth integrals and positive homogeneity.
- 3.9. Parametric integrals.
- 4. Vectorial integrals: Q-, P-, R-convexity.
- 4.1. The Euler-Lagrange differential equation.
- 4.2. Lower semicontinuity in the scalar case implies convexity.
- 4.3. Q-, P- and R-convexity
- 4.4. Q-convexity implies R-convexity.
- 4.5. Lower semicontinuity implies Q-convexity.

#### Back

#### Advanced Topics in Sampling (MAT11702D)



#### Advanced Topics in Operation Research (MAT11703D)

- 1. Optimization with Genetic Algorithms{\}newline
- 2. Networks and Graphs{\}newline
- 3. Dynamic programming{\}newline
- 4. Project Management{\}newline
- 5. Productivity and Efficiency Analysis{\}newline
- 6. Markov Decision Processes

#### Back

#### Advanced Topics in Stochastic Processes (MAT11704D)

- 1. Poisson processes and its variants.
- 2. Renewal processes and its variants...
- 3. Networks of queues and applications to the modeling of telecommunications systems.
- 4. Diffusion process and Wiener process, Brownian motion.
- 5. Ito and Stratonovich stochastic integrals, Ito's formula.
- 6. Stochastic differential equations and its application to modeling animal population growth and financial data.

#### Back

#### **Topics in Computational Statistics (MAT11705D)**

- 1. Introduction to the R language.
- 2. Generation of Pseudo-Random Numbers.
- 3. Monte Carlo Methods in Statistical Inference.
- 4. Resampling methods: Bootstrap, Jackknife.
- 5. Markov Chain Monte Carlo methods (MCMC).
- 6. ML estimation and the EM algorithm.

#### Back

#### Advanced Topics in Experimental Delineation (MAT11706D)

1. Theory and practice of experimental design. Complete and balanced incomplete block designs. Latin square designs.

2. Factorial designs and fractional factorial designs.

3. Split-plot designs. Split-Block designs. Repeated measures and their relationship with the Split-Plot designs and Split Block designs.

- 4. Lattice Designs.
- 5. Crossover Designs.
- 6. Response Surface Methods.

#### Back

#### Advanced Topics in Multivariate Statistic (MAT11707D)

1. Multivariate Distributions (multivariate normal distribution, Wishart distribution, Hotelling distribution,

- the Wilks Lambda statistic).
- 2.Methods of Analysis Interdependence
- 3.Independent Conponente Analysis
- 4. Methods of Analysis Dependence
- 5. Multidimensional Scaling
- 6.Date Mining



#### Topics of Space-Time Modeling (MAT11708D)

- 1. Brief review of the essential concepts of Stochastic processes.
- 2. Temporal linear models: SARIMA model
- 3. Spatial point processes
- 4. Continuous spatial models: kriging and co-kriging spatial interpolation methods
- 5. Spatial clustering analysis
- 6. Analysis of time series and spatial data using the software R.

#### Back

#### Tópics in Analysis of Categorical Data (MAT11709D)

#### Back

#### Topics of Statistical Modelling (MAT11710D)

- 1. Quality control charts.
- 2. Process capability analysis and six-sigma methodology.
- 3. Inspection systems policies.
- 4. Survival models.

#### Back

#### Structural Equation Models (MAT10180D)

1. Introduction in Structural Equation Modeling (SEM). SEM and other multivariate techniques.Basic concepts and applications of SEM . SEM in research. Brief history of SEM.

2. Univariate and multivariate linear regression models. Exploratory Factorial Analysis. Path Analysis. Moderation effects and mediation effects with latent variables.

3. Measurement model and  $1^{\circ}$  and  $2^{\circ}$  order Confirmatory Factorial Analysis. Assessing measurement model validity. Construct validity.

4. Causal relationships and latent variables. Stages in testing structural theory: specification, identification, estimation, validation and modification of models. Recoursive and non recoursive models. Multigroup analysis. Bootstrapping in SEM.

5. Extensions. Introduction to Latent Curve Models with longitudinal data.