Study Plan

School: School of Sciences and Technology
Degree: Master
Course: Mathematics and Applications (cód. 594)

Specialization *** TRANSLATE ME: Álgebra e Análise ***

1st Year - 1st Semester
Specialization * TRANSLATE ME: Álgebra e Análise *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Free Options					
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10139M	Algebra	Mathematics	6	Semester	156
MAT11917M	Complements of Numerical Analysis	Mathematics	6	Semester	156
MAT10138M	Mathematical Logic	Mathematics	6	Semester	156
MAT10151M	Theory of Ordinary Differential Equations	Mathematics	6	Semester	156

1st Year - 2nd Semester
Specialization * TRANSLATE ME: Álgebra e Análise *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11919M	Seminary I	Mathematics	6	Semester	156
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11918M	Combinatoric	Mathematics	6	Semester	156
MAT10152M	Numerical Optimization	Mathematics	6	Semester	156
MAT10158M	Dynamical Systems	Mathematics	6	Semester	156
MAT10155M	Theory of Partial Differential Equations	Mathematics	6	Semester	156

2nd Year - 3rd Semester
Specialization * TRANSLATE ME: Álgebra e Análise *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10165M	Qualitative Methods in Nonlinear Differential Equations	Mathematics	6	Semester	156
MAT10154M	Topics of Functional Analysis	Mathematics	6	Semester	156
MAT10141M	Elements of Cryptography	Mathematics	6	Semester	156
MAT10143M	Axiomatic Set Theory	Mathematics	6	Semester	156

Dissertation
Internship

2nd Year - 4th Semester
Specialization * TRANSLATE ME: Álgebra e Análise *****

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
Internship					

Specialization Statistics

1st Year - 1st Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10167M	Experimental Design	Mathematics	6	Semester	156
MAT10168M	Computational Statistics	Mathematics	9	Semester	234
MAT10169M	Statistical Inference	Mathematics	9	Semester	234
MAT10170M	Stochastic Processes	Mathematics	6	Semester	156

Group of Free Options

1st Year - 2nd Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT11919M	Seminary I	Mathematics	6	Semester	156

1st Year - 2nd Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					
	Component code	Name	Scientific Area Field	ECTS	Duration	Hours

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10171M	Categorical Data Analysis	Mathematics	9	Semester	234
MAT10172M	SStochastic Differential Equations and Biological Applications	Mathematics	6	Semester	156
MAT10173M	Multivariate Data Statistics	Mathematics	9	Semester	234
MAT10174M	Time Series	Mathematics	6	Semester	156

2nd Year - 3rd Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10175M	Sampling Biological Populations	Mathematics	6	Semester	156
MAT10176M	Quality Control and Reliability	Mathematics	6	Semester	156
MAT10177M	Operational Research	Mathematics	6	Semester	156
MAT10178M	Mathematical Models in Biology	Mathematics	6	Semester	156

Dissertation	
Internship	

2nd Year - 4th Semester
Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
*** TRANSLATE ME:Relatório de Estágio ***					
Internship					

Specialization Mathematics and Applications

1st Year - 1st Semester
Specialization Mathematics and Applications

Component code	Name	Scientific Area Field	ECTS	Duration	Hours			
Group of Options								
Component code	Name		Scientific Area Field	ECTS	Duration	Hours		
MAT10139M	Algebra		Mathematics	6	Semester	156		
MAT11917M	Complements of Numerical Analysis		Mathematics	6	Semester	156		
MAT10167M	Experimental Design		Mathematics	6	Semester	156		
MAT10168M	Computational Statistics		Mathematics	9	Semester	234		
MAT10169M	Statistical Inference		Mathematics	9	Semester	234		
MAT10138M	Mathematical Logic		Mathematics	6	Semester	156		
MAT10170M	Stochastic Processes		Mathematics	6	Semester	156		
MAT10151M	Theory of Ordinary Differential Equations		Mathematics	6	Semester	156		
Group of Free Options								

1st Year - 2nd Semester
Specialization Mathematics and Applications

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10171M	Categorical Data Analysis	Mathematics	9	Semester	234
MAT11918M	Combinatoric	Mathematics	6	Semester	156
MAT10172M	SStochastic Differential Equations and Biological Applications	Mathematics	6	Semester	156
MAT10173M	Multivariate Data Statistics	Mathematics	9	Semester	234
MAT10152M	Numerical Optimization	Mathematics	6	Semester	156
MAT10174M	Time Series	Mathematics	6	Semester	156
MAT10158M	Dynamical Systems	Mathematics	6	Semester	156
MAT10155M	Theory of Partial Differential Equations	Mathematics	6	Semester	156
Group of Free Options					
MAT11919M	Seminary I	Mathematics	6	Semester	156

2nd Year - 3rd Semester
Specialization Mathematics and Applications

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Group of Options					

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10175M	Sampling Biological Populations	Mathematics	6	Semester	156
MAT10176M	Quality Control and Reliability	Mathematics	6	Semester	156
MAT10141M	Elements of Cryptography	Mathematics	6	Semester	156
MAT10177M	Operational Research	Mathematics	6	Semester	156
MAT10165M	Qualitative Methods in Nonlinear Differential Equations	Mathematics	6	Semester	156
MAT10178M	Mathematical Models in Biology	Mathematics	6	Semester	156
MAT10143M	Axiomatic Set Theory	Mathematics	6	Semester	156
MAT10154M	Topics of Functional Analysis	Mathematics	6	Semester	156

Dissertation					
Internship					

2nd Year - 4th Semester
Specialization Mathematics and Applications

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
Internship					

Conditions for obtaining the Degree:

*** TRANSLATE ME: Dos 120 ECTS necessários para a obtenção do grau de Mestre em cada área de especialização o aluno terá de fazer:

a)   72 ECTS em unidades curriculares, das quais 60 ECTS têm de ser na área de especialização;

b)   48 ECTS para a Dissertação Científica ou Estágio Profissionalizante.

Os 60 ECTS da área de especialização deverão ser obtidos do seguinte modo:

1) Pelo menos 30 ECTS em unidades curriculares do 1º ano (excluindo a unidades curriculares de Seminário I) e pelo menos 12 ECTS no 2º ano, perfazendo um total de 60 ECTS, na área que respeita ao perfil escolhido;

2) 12 ECTS em quaisquer unidades curriculares das 3 áreas de Especialização. ***

Program Contents

Algebra (MAT10139M)
Rings and modules. Fields and extensions of fields. Splitting fields. Galois theory. Ring extensions. Transcendental extensions. Hilbert's Nullstellentzatz. Algebraic spaces.

Complements of Numerical Analysis (MAT11917M)
Methods for solving large linear systems.
Methods for solving nonlinear systems.
Methods of calculation of eigenvalues and eigenvectors.
ODEs: initial value problems. Rigid systems.
ODEs: boundary value problems.
EDP's: finite differences, finite elements.

Mathematical Logic (MAT10138M)
1. Propositional logic
1.1. Syntax and Semantics. Deduction and logical implication.
1.2. Metatheorems of Soundness and Completeness, consistency, compatibility.
1.3. Fundamental properties of Propositional logic: Interpolation, compactness, decidibility.
2. First-order logic
2.1. Syntax, deduction. Semantics, models, logical implication.
2.2. Metatheorems of Soundness and Completeness, compactness and the Theorem of Löwenheim-Skolem.
2.3. Applications: formal and informal reasoning, nonstandard models of arithmetic.

Theory of Ordinary Differential Equations (MAT10151M)
Bounded variation functions and absolutely continuous functions.
Carathéodory type solutions of ODES in finite-dimensional spaces .
Peano's Theorem. Extension of solutions.
Topological properties of the solution set.
Uniqueness of solution.
Picard-Lindelof method.
Ordinary differential equations in Banach spaces.
Equations in infinite dimension, with an unbounded linear operator.
Weak solutions.
Invariance and viability.

Seminary I (MAT11919M)
Teachers of DMAT and researchers of the CIMA-UE, will be invited, preferably but not exclu-sively, to share their work and / or research areas.

Combinatoric (MAT11918M)
Elements of generating functions
Graphs
Oriented Matroids
Advanced topics

Numerical Optimization (MAT10152M)
1. Elements of Convex Analysis. Necessary and sufficient conditions of optimality.
2. Nonlinear Optimization. One-dimensional optimization. Unconstrained optimization with and without derivatives.
3. Constrained Optimization. Penalty functions. Interior point method.
4. Multiobjective Optimization. Global Optimization. Evolutionary and Genetic Algorithms.
5. Dynamic Programming. Applications to the Optimal Control problems. Automatic Differentiation.

Dynamical Systems (MAT10158M)
Functions of the interval in the interval and the circle in the circle: hiperbolicity, symbolic dynamics, topological conjugation, theorem of Sharkovsky, structural stability, topological bifurcation, invariants, renormalization, chaos, theory of the kneading of Milnor and Thurston.
Iteration of complex functions: normal families, periodic, Julia joint points, sets of Mandelbrot.
Applications.

Theory of Partial Differential Equations (MAT10155M)
Phenomenology and modelling of the Heat Equation.
Classification of PDEs and canonical forms.
Series and Fourier transform. Applications.
Solutions of the Heat Equation.
The Burgers Equation.
Variational methods.
Energy and entropy methods.
Main work options:
a) Financial Mathematics (Black-Scholes Eq.);
b) Applications to Biology (Transport Eqs.);
c) Numerical Analysis (Hilbert-Huang Transform).

Qualitative Methods in Nonlinear Differential Equations (MAT10165M)
1. Variational Methods: Deformation Theorem and Palais-Smale condition. Min-max theorems. Mountain Pass Theorem. Saddle points. Link Theorems.
2. Topological Degree: degree theory for continuous functions. Degree in finite dimension: Brouwer's Degree. Degree in infinite dimension: Leray-Schauder's degree. Degree for compact perturbations of a linear operator: Degree of Coincidence. Applications to differential equations. Fixed point theorems. Applications to partial differential equations.
3. Boundary Value Problems : Method of Upper and Lower-solutions: direct and monotone iterative methods. Maximum principle and Comparison Theorems. Existence of extremal solutions. Non- ordered Lower and upper-solutions for higher order problems.

Topics of Functional Analysis (MAT10154M)
To choose material among the following topics:
1. Theory of distributions. Sobolev spaces. Embedding theorems.
2. Semigroups of linear operators. Hille-Yosida Theorem. Monotone operators.
3. Leray-Schauder Theory of topological degree. Nonlinear operators. Fixed points.
4. Spectral Theory of linear operators in Hilbert spaces.

Last update 20/04/2024

Elements of Cryptography (MAT10141M)
Integers
Congruences and Residue Class Rings
Encryption
Probability
DES
Public-Key
Discret Logarithms
Hash Functions
Digital Signatures
Finite Fields
Elliptic Curves

Axiomatic Set Theory (MAT10143M)
1. The language and axioms of Zermelo-Fraenkel and elementary consequences.
2. Well-orderings and von Neumann ordinals. Transfinite induction and recursion.
Ordinal arithmetic.
3. The cumulative hierarchy.
4. Numerability, non-numerability. Cardinals and cardinal arithmetic. The continuum problem. Perfect sets.
Theorem of Cantor-Bendixon.
5. Axiom of Choice and some consequences.
6. Continuum Hypothesis.

Experimental Design (MAT10167M)
Scientific method and experimental design.
Analysis of variance models: fixed effects (single and multiple factor), random effects (single and multiple factor) and mixed effects.
Split-plot and nested designs.
Multiple comparisons.
Complete and incomplete block designs. Latin square designs.
Non-parametric approaches.
Simple linear regression model and multiple regression model (estimation, inference, prediction, model adequacy and validation).
Diagnostics for influence points, outliers,
multicollinearity and autocorrelation. Model selection.
Analysis of Covariance.
Nonlinear Regression.

Last update 20/04/2024

Computational Statistics (MAT10168M)
1. Statistical modelling. Common Statistical models. Adjustment non-parametric tests. Independence tests and uniformity tests. Graphics methods.
2. Maximum Likelihood estimation and the EM algorithm (with resource to numerical methods).
3. Uniform pseudorandom numbers generaton.
4. Pseudorandom numbers generation with a specified distribution.
5. Resampling methods.
6. Monte Carlo Method.
7. Bootstrap and Jackknife.
8. Markov Chains Monte Carlo Methods (MCMC), Gibbs algorithm and Metroplolis-Hasting algorithm.
9. Applications and use of statistical software.

Statistical Inference (MAT10169M)
Fundamental concepts of probability (measure and probability, random vectors, marginal and conditional distributions, expected values, generating and characteristics functions, functions of random vectors and transformations).
Review of discrete and continuous distributions properties. Exponential families.
Multinormal and multinomial distributions.
Stochastic convergences and limit theorems.
Sampling and the most used sampling distributions.
Point estimation. Estimation methods (moments, maximum likelihood, least squares and bayes estimators). Properties of estimators. Crámer-Rao lower bound. Asymptotic behaviour. Robustness.
Interval estimation. Methods for finding interval estimators. Properties. Classical and bayesian approach.
Hypotheses testing. Type I and Type II probability errors. Duality. Methods for finding testes. Likelihood ratio tests. Properties of tests. Neyman-Pearson theorem, most powerful tests. Asymptotic behaviour. Robustness. Classical and bayesian approach.

Stochastic Processes (MAT10170M)
1. General concepts of Stochastic Processes.
2. Martingale and applications.
3. Markov chains in discrete time.
4. General concepts of time series.
5. Poisson process of homogeneous and inhomogeneous.
6. Compound Poisson process.
7. Processes of birth and death.
8. Introduction to queues.
9. Renewal processes.
10. Methods of Monte Carlo simulation.

Categorical Data Analysis (MAT10171M)
Contingency Tables.
Generalized linear models: characterization, link functions, statistical modelling, assumptions, residual analysis, validation and inference.
Discrete and continuous models: Logistic (Binomial, Ordinal and Multinomial), Poisson, Negative Binomial, Inverse-Gaussian, Gama, Lognormal.
Generalized Estimating Equations (GEE).
Other topics in statistical modeling of categorical data.

SStochastic Differential Equations and Biological Applications (MAT10172M)

Module 1. Introduction to SDE and Applications:

Wiener Process and diffusions

Martingales, adapted processes

Stochastic integrals, sketch of the construction of the Itô integral, and Itô's Theorem

Existence and Uniqueness theorem for SDE

Strong and weak solutions

Formula of Feynman-Kac.

Module 2. Biological Applications of SDE:

The Stratonovich integral, relations with the Itô integral and their use in applications

Biological applications in population dynamics and growth of living organisms or biological tissues in a random environment

Study of extinction and extinction times. Existence of stationary densities. Qualitative and quantitative study of solutions (by simulation if required)

Optimization problems in the management of renewable natural resources

Comparison with models based on birth and death processes (demographic randomness) and approximation of these models by SDEs

Applications to population genetics

Statistical issues in SDEs (estimation and prediction).

Multivariate Data Statistics (MAT10173M)

1. Overview of Multivariate Statistical Methods. Introduction. Dependence Techniques and Interdependence Techniques. Extentions.
2. Preliminary and exploratory multivariate data analysis
3. Principal Component Analysis
4. Exploratory Factorial Analysis versus Confirmatory Factorial Analysis
5. Cluster Analysis
6. Discriminant Analysis
7. Structural Equation Modeling: an introduction

Time Series (MAT10174M)

1. Brief review of the essential concepts of Stochastic processes. Identification of temporal patterns. Temporal Decomposition Models
2. Linear models: ARMA, ARIMA and SARIMA
3. Non linear models: ARCH and GARCH
4. Temporal regression models
5. Analysis and modelling of time series, using the above methods, using the software R.

Sampling Biological Populations (MAT10175M)

1. Elements of Statistical Inference and finite population sampling.
2. Estimation of wildlife population abundance.
3. Distance sampling, capture-recapture and combined models.
4. Estimation of demographic parameters (survival, recruitment, transition probabilities, migration rates).
5. Parameter estimation in Community Dynamics.

Quality Control and Reliability (MAT10176M)
Control charts for variables and attributes.
Process capability analysis. 6-sigma processes.
Capacity of the measuring system. Accuracy and Precision. Repeatability and reproducibility.
Acceptance sampling. Different sampling plans. MIL STD tables.
Sampling methods in quality control.
Reliability and survival.
Series and parallel systems.
Parametric and non-parametric hazard models.
Inspection systems policies.

Operational Research (MAT10177M)
1. Linear and Nonlinear Programming: Applications, Revised Simplex; Interior Point methods. Integer and Mixed Linear Programming: Applications, Branch and Bound Method. Nonlinear Programming: Applications, Karush-KuhnTucker Conditions (KKT), Evolutionary and Genetic Methods.
2. Optimization over Networks and Graphs; Inventory theory and Project management: graphs: applications, definitions, Matrix representation. Trees. Facility location and maximum flux problems. Project Management (PERT/CPM). Basics of Inventory Theory.
3. Decision Support Systems: Decision Trees. Utility Functions. Multi-Criteria Analysis: Multi-Attribute, Multi-Objective. Game theory.

Mathematical Models in Biology (MAT10178M)
1. Introduction to population and ecosystem modelling.
2. Deterministic and stochastic mathematical population growth models.
3. Introduction to population genetic modelling.
4. Natural resources modelling.
5. Structured population modelling.
6. Demographic models.
7. Spatial dispersion models.
8. Ecosystem modelling (competition, predation, etc.)
9. Deterministic epidemic modelling.