

# Study Plan

School: School of Sciences and Technology

Degree: Master

Course: Mathematics and Applications (São Tomé e Príncipe) (cód. 632)

# Specialization \*\*\* TRANSLATE ME: Álgebra e Análise \*\*\*

# 1st Year - 1st Semester

Specialization \*\*\* TRANSLATE ME: Álgebra e Análise \*\*\*

omponent code	Name	Scientific Area Field	ECTS	Duration		Hours		
roup of Free Optior	าร							
roup of Options								
Component code		Name			Scientific Area Field	ECTS	Duration	Hours
	Algeb	ra			Mathematics	6	Semester	156
MAT10139M								
	Comp	lements of Numerical An	alysis		Mathematics	6	Semester	156
MAT11917M								
	Mathe	ematical Logic			Mathematics	6	Semester	156
MAT10138M								
	Theor	y of Ordinary Differentia	l Equation	าร	Mathematics	6	Semester	156
MAT10151M		-	•					

# 1st Year - 2nd Semester

Specialization \*\*\* TRANSLATE ME: Álgebra e Análise \*\*\*

omponent code	Name		Scientific Area Fi	ield	EC.	TS Dura	ntion	Н
	Seminary I		Mathematics		6	Seme	ester	156
AT11919M								
roup of Options						'		
Component code	Name	Sci	entific Area Field	EC	CTS	Duration	Ho	ours
	Combinatoric	Ma	thematics	6		Semester	156	j
MAT11918M								
	Numerical Optimization	Ma	thematics	6		Semester	156	j
MAT10152M								
	Dynamical Systems	Ma	thematics	6		Semester	156	j
MAT10158M								
	Theory of Partial Differential Equations	Ma	thematics	6		Semester	156	j
MAT10155M								



## 2nd Year - 3rd Semester

Specialization \*\*\* TRANSLATE ME: Álgebra e Análise \*\*\*

omponent code	Name	Scientific Area Field	ECTS	Duration		Hours		
Froup of Options								
Component code	!	Name			Scientific Area Field	ECTS	Duration	Hours
MAT10165M	Qualitions	tative Methods in Nonline	ear Differ	ential Equa-	Mathematics	6	Semester	156
MAT10154M	Topic	s of Functional Analysis			Mathematics	6	Semester	156
MAT10141M	Eleme	ents of Cryptography			Mathematics	6	Semester	156
MAT10143M	Axion	natic Set Theory			Mathematics	6	Semester	156
Dissertation Internship								

# 2nd Year - 4th Semester

Specialization \*\*\* TRANSLATE ME: Álgebra e Análise \*\*\*

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
Internship					

# **Specialization Statistics**

# 1st Year - 1st Semester Specialization Statistics

Component code	Name	Scientific Area Field	ECTS	Duration		Hours		
Group of Options								
Component code		Name			Scientific Area Field	ECTS	Duration	Hours
MAT10167M	Exper	imental Design			Mathematics	6	Semester	156
MAT10168M	Comp	outational Statistics			Mathematics	9	Semester	234
MAT10169M	Statis	tical Inference			Mathematics	9	Semester	234
MAT10170M	Stoch	astic Processes			Mathematics	6	Semester	156

# 1st Year - 2nd Semester

**Specialization Statistics** 

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
	Seminary I	Mathematics	6	Semester	156
MAT11919M					



# 1st Year - 2nd Semester Specialization Statistics

omponent code	Name		Scientific Area Fie		ECT	S Dura	tion	Ho
roup of Options						•		
Component code	Name	Scie	entific Area Field	EC	TS	Duration	Ho	urs
	Categorical Data Analysis	Mat	hematics	9		Semester	234	
MAT10171M								
	SStochastic Differential Equations and Biological	Mat	hematics	6		Semester	156	
MAT10172M	Applications							
	Multivariate Data Statistics	Mat	hematics	9		Semester	234	
MAT10173M								
	Time Series	Mat	hematics	6		Semester	156	
MAT10174M								

# 2nd Year - 3rd Semester Specialization Statistics

omponent code   1	Name	Scientific Area Field	ECTS	Duration		Hours		
roup of Options								
Component code		Name			Scientific Area Field	ECTS	Duration	Hours
MAT10175M	Sampl	ling Biological Population	ns		Mathematics	6	Semester	156
MAT10176M	Qualit	y Control and Reliability			Mathematics	6	Semester	156
MAT10177M	Opera	tional Research			Mathematics	6	Semester	156
MAT10178M	Mathe	ematical Models in Biolo	gy		Mathematics	6	Semester	156

# 2nd Year - 4th Semester Specialization Statistics

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Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
*** TRANSLATE N	ЛЕ:Relató	rio de Estágio ***			
Internship					

# **Specialization Mathematics and Applications**



1st Year - 1st Semester

**Specialization Mathematics and Applications** 

Component code		Name			Scientific Area Field	ECTS	Duration	Hours
MAT10139M	Algebra				Mathematics	6	Semester	156
MAT11917M	Complements of Nu	merical Ana	lysis		Mathematics	6	Semester	156
MAT10167M	Experimental Design				Mathematics	6	Semester	156
MAT10168M	Computational Statistics			Mathematics	9	Semester	234	
MAT10169M	Statistical Inference				Mathematics	9	Semester	234
MAT10138M	Mathematical Logic				Mathematics	6	Semester	156
MAT10170M	Stochastic Processe	5			Mathematics	6	Semester	156
MAT10151M	Theory of Ordinary	Differential	Equations	5	Mathematics	6	Semester	156

# 1st Year - 2nd Semester

**Specialization Mathematics and Applications** 

mponent code	Name	Scientific Area F	ield E	CTS Dura	tion Ho
oup of Options				•	
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
	Categorical Data Analysis	Mathematics	9	Semester	234
MAT10171M					
	Combinatoric	Mathematics	6	Semester	156
MAT11918M					
	SStochastic Differential Equations and Biological	Mathematics	6	Semester	156
MAT10172M	Applications				
	Multivariate Data Statistics	Mathematics	9	Semester	234
MAT10173M					
	Numerical Optimization	Mathematics	6	Semester	156
MAT10152M					
	Time Series	Mathematics	6	Semester	156
MAT10174M					
	Dynamical Systems	Mathematics	6	Semester	156
MAT10158M					
MAT40455M	Theory of Partial Differential Equations	Mathematics	6	Semester	156
MAT10155M					
oup of Free Option					
	Seminary I	Mathematics	6	Seme	ster   156
AT11919M					



# 2nd Year - 3rd Semester

## **Specialization Mathematics and Applications**

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10175M	Sampling Biological Populations	Mathematics	6	Semester	156
MAT10176M	Quality Control and Reliability	Mathematics	6	Semester	156
MAT10141M	Elements of Cryptography	Mathematics	6	Semester	156
MAT10177M	Operational Research	Mathematics	6	Semester	156
MAT10165M	Qualitative Methods in Nonlinear Differential Equations	Mathematics	6	Semester	156
MAT10178M	Mathematical Models in Biology	Mathematics	6	Semester	156
MAT10143M	Axiomatic Set Theory	Mathematics	6	Semester	156
MAT10154M	Topics of Functional Analysis	Mathematics	6	Semester	156

### 2nd Year - 4th Semester

**Specialization Mathematics and Applications** 

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
Dissertation					
Internship					

# **Program Contents**

#### Back

# Algebra (MAT10139M)

Rings and modules. Fields and extensions of fields. Splitting fields. Galois theory. Ring extensions. Transcendental extensions. Hilbert's Nullstellentzatz. Algebraic spaces.

#### Back

# Complements of Numerical Analysis (MAT11917M)

Methods for solving large linear systems. Methods for solving nonlinear systems.

Methods of calculation of eigenvalues and eigenvectors.

ODEs: initial value problems. Rigid systems.

ODEs: boundary value problems.

EDP's: finite differences, finite elements.



#### Mathematical Logic (MAT10138M)

- 1. Propositional logic
- 1.1. Syntax and Semantics. Deduction and logical implication.
- 1.2. Metatheorems of Soundness and Completeness, consistency, compatibility.
- 1.3. Fundamental properties of Propositional logic: Interpolation, compactness, decidibility.
- 2. First-order logic
- 2.1. Syntax, deduction. Semantics, models, logical implication.
- 2.2. Metatheorems of Soundness and Completeness, compactness and the Theorem of Löwenheim-Skolem
- 2.3. Applications: formal and informal reasoning, nonstandard models of arithmetic.

#### Back

# Theory of Ordinary Differential Equations (MAT10151M)

Bounded variation functions and absolutely continuous functions.

Carathéodory type solutions of ODES in finite-dimensional spaces .

Peano's Theorem. Extension of solutions.

Topological properties of the solution set.

Uniqueness of solution.

Picard-Lindelof method.

Ordinary differential equations in Banach spaces.

Equations in infinite dimension, with an unbounded linear operator.

Weak solutions.

Invariance and viability.

#### Back

## Seminary I (MAT11919M)

Teachers of DMAT and researchers of the CIMA-UE, will be invited, preferably but not exclu-sively, to share their work and / or research areas.

#### Back

### Combinatoric (MAT11918M)

Elements of generating functions Graphs Oriented Matroids Advanced topics

#### Back

#### **Numerical Optimization (MAT10152M)**

- 1. Elements of Convex Analysis. Necessary and sufficient conditions of optimality.
- 2. Nonlinear Optimization. One-dimensional optimization. Unconstrained optimization with and without derivatives.
- 3. Constrained Optimization. Penalty functions. Interior point method.
- 4. Multiobjective Optimization. Global Optimization. Evolutionary and Genetic Algorithms.
- 5. Dynamic Programming. Applications to the Optimal Control problems. Automatic Differentiation.



#### **Dynamical Systems (MAT10158M)**

Functions of the interval in the interval and the circle in the circle: hiperbolicity, symbolic dynamics, topological conjugation, theorem of Sharkovsky, structural stability, topological bifurcation, invariants, renormalization, chaos, theory of the kneading of Milnor and Thurston. Iteration of complex functions: normal families, periodic, Julia joint points, sets of Mandelbrot. Applications.

#### Back

#### Theory of Partial Differential Equations (MAT10155M)

Phenomenology and modelling of the Heat Equation.

Classification of PDEs and canonical forms.

Series and Fourier transform. Applications.

Solutions of the Heat Equation.

The Burgers Equation.

Variational methods.

Energy and entropy methods.

Main work options:

- a) Financial Mathematics (Black-Scholes Eq.);
- b) Applications to Biology (Transport Eqs.);
- c) Numerical Analysis (Hilbert-Huang Transform).

#### Back

### Qualitative Methods in Nonlinear Differential Equations (MAT10165M)

- 1. Variational Methods: Deformation Theorem and Palais-Smale condition. Min-max theorems. Mountain Pass Theorem. Saddle points. Link Theorems.
- 2. Topological Degree: degree theory for continuous functions. Degree in finite dimension: Brouwer's Degree. Degree in infinite dimension: Leray-Schauder's degree. Degree for compact perturbations of a linear operator: Degree of Coincidence. Applications to differential equations. Fixed point theorems. Applications to partial differential equations.
- 3. Boundary Value Problems: Method of Upper and Lower-solutions: direct and monotone iterative methods. Maximum principle and Comparison Theorems. Existence of extremal solutions. Non- ordered Lower and upper-solutions for higher order problems.

#### Back

### Topics of Functional Analysis (MAT10154M)

To choose material among the following topics:

- 1. Theory of distributions. Sobolev spaces. Embedding theorems.
- 2. Semigroups of linear operators. Hille-Yosida Theorem. Monotone operators.
- 3. Leray-Schauder Theory of topological degree. Nonlinear operators. Fixed points.
- 4. Spectral Theory of linear operators in Hilbert spaces.



## Elements of Cryptography (MAT10141M)

Integers

Congruences and Residue Class Rings

Encryption

Probability

DES

Public-Key

Discret Logarithms

Hash Functions

Digital Signatures

Finite Fields

Elliptic Curves

#### Back

## Axiomatic Set Theory (MAT10143M)

- 1. The language and axioms of Zermelo-Fraenkel and elementary consequences.
- 2. Well-orderings and von Neumann ordinals. Transfinite induction and recursion.

Ordinal arithmetic.

- 3. The cumulative hierarchy.
- 4. Numerability, non-numerability. Cardinals and cardinal arithmetic. The continuum problem. Perfect sets.

Theorem of Cantor-Bendixon.

- 5. Axiom of Choice and some consequences.
- 6. Continuum Hypothesis.

#### Back

## Experimental Design (MAT10167M)

Scientific method and experimental design.

Analysis of variance models: fixed effects (single and multiple factor), random effects (single and multiple factor) and mixed effects.

Split-plot and nested designs.

Multiple comparisons.

Complete and incomplete block designs. Latin square designs.

Non-parametric approaches.

Simple linear regression model and multiple regression model (estimation, inference, prediction, model adequacy and validation). Diagnostics for influence points, outliers,

multicollinearity and autocorrelation. Model selection.

Analysis of Covariance.

Nonlinear Regression.



#### Computational Statistics (MAT10168M)

- 1. Statistical modelling. Common Statistical models. Adjustment non-parametric tests. Independence tests and uniformity tests. Graphics methods.
- 2. Maximum Likelihood estimation and the EM algorithm ( with resource to numerical methods).
- 3. Uniform pseudorandom numbers generaton.
- 4. Pseudorandom numbers generation with a specified distribution.
- 5. Resampling methods.
- 6. Monte Carlo Method.
- 7. Bootstrap and Jackknife.
- 8. Markov Chains Monte Carlo Methods (MCMC), Gibbs algorithm and Metroplolis-Hasting algorithm.
- 9. Applications and use of statistical software.

#### Back

# Statistical Inference (MAT10169M)

Fundamental concepts of probability (measure and probability, random vectors, marginal and conditional distributions, expected values, generating and characteristics functions, functions of random vectors and transformations).

Review of discrete and continuous distributions properties. Exponential families.

Multinormal and multinomial distributions.

Stochastic convergences and limit theorems.

Sampling and the most used sampling distributions.

Point estimation. Estimation methods (moments, maximum likelihood, least squares and bayes estimators). Properties of estimators. Crámer-Rao lower bound. Asymptotic behaviour. Robustness. Interval estimation. Methods for finding interval estimators. Properties. Classical and bayesian approach. Hypotheses testing. Type I and Type II probability errors. Duality. Methods for finding testes. Likelihood ratio tests. Properties of tests. Neyman-Pearson theorem, most powerful tests. Asymptotic behaviour. Robustness. Classical and bayesian approach.

### Back

## Stochastic Processes (MAT10170M)

- 1. General concepts of Stochastic Processes.
- 2. Martingale and applications.
- 3. Markov chains in discrete time.
- 4. General concepts of time series.
- 5. Poisson process of homogeneous and inhomogeneous.
- 6. Compound Poisson process.
- 7. Processes of birth and death.
- 8. Introduction to gueues.
- 9. Renewal processes.
- 10. Methods of Monte Carlo simulation.

#### Back

### Categorical Data Analysis (MAT10171M)

Contingency Tables.

Generalized linear models: characterization, link functions, statistical modelling, assumptions, residual analysis, validation and inference.

Discrete and continuous models: Logistic (Binomial, Ordinal and Multinomial), Poisson, Negative Binomial, Inverse-Gaussian, Gama, Lognormal.

Generalized Estimating Equations (GEE).

Other topics in statistical modeling of categorical data.



## SStochastic Differential Equations and Biological Applications (MAT10172M)

Module 1. Introduction to SDE and Applications:

Wiener Process and diffusions

Martingales, adapted processes

Stochastic integrals, sketch of the construction of the Itô integral, and Itô's Theorem

Existence and Uniqueness theorem for SDE

Strong and weak solutions

Formula of Feynman-Kac.

### Module 2. Biological Applications of SDE:

The Stratonovich integral, relations with the Itô integral and their use in applications

Biological applications in population dynamics and growth of living organisms or biological tissues in a random environment Study of extinction and extinction times. Existence of stationary densities. Qualitative and quantitative study of solutions (by simulation if required)

Optimization problems in the management of renewable natural resources

Comparison with models based on birth and death processes (demographic randomness) and approximation of these models by SDEs

Applications to population genetics

Statistical issues in SDEs (estimation and prediction).

#### Back

# Multivariate Data Statistics (MAT10173M)

- 1. Overview of Multivariate Statistical Methods. Introduction. Dependence Techniques and Interdependence Techniques. Extentions.
- 2. Preliminary and exploratory multivariate data analysis
- 3. Principal Component Analysis
- 4. Exploratory Factorial Analysis versus Confirmatory Factorial Analysis
- 5. Cluster Analysis
- 6. Discriminant Analysis
- 7. Structural Equation Modeling: an introduction

#### Rack

#### Time Series (MAT10174M)

- 1. Brief review of the essential concepts of Stochastic processes. Identification of temporal patterns. Temporal Decomposition Models
- 2. Linear models: ARMA, ARIMA and SARIMA
- 3. Non linear models: ARCH and GARCH
- 4. Temporal regression models
- 5. Analysis and modelling of time series, using the above methods, using the software R.

## Back

# Sampling Biological Populations (MAT10175M)

- 1. Elements of Statistical Inference and finite population sampling.
- 2. Estimation of wildlife population abundance.
- 3. Distance sampling, capture-recapture and combined models.
- 4. Estimation of demographic parameters (survival, recruitment, transition probabilities, migration rates).
- 5. Parameter estimation in Community Dynamics.



## Quality Control and Reliability (MAT10176M)

Control charts for variables and attributes.

Process capability analysis. 6-sigma processes.

Capacity of the measuring system. Accuracy and Precision. Repeatability and reproducibility.

Acceptance sampling. Different sampling plans. MIL STD tables.

Sampling methods in quality control.

Reliability and survival.

Series and parallel systems.

Parametric and non-parametric hazard models.

Inspection systems policies.

#### Back

#### Operational Research (MAT10177M)

- 1. Linear and Nonlinear Programming: Applications, Revised Simplex; Interior Point methods. Integer and Mixed Linear Programming: Applications, Branch and Bound Method. Nonlinear Programming: Applications, Karush-KuhnTucker Conditions (KKT), Evolutionary and Genetic Methods.
- 2. Optimization over Networks and Graphs; Inventory theory and Project management: graphs: applications, definitions, Matrix representation. Trees. Facility location and maximum flux problems. Project Management (PERT/CPM). Basics of Inventory Theory.
- 3. Decision Support Systems: Decision Trees. Utility Functions. Multi-Criteria Analysis: Multi-Attribute, Multi-Objective. Game theory.

#### Back

### Mathematical Models in Biology (MAT10178M)

- 1. Introduction to population and ecosystem modelling.
- 2. Deterministic and stochastic mathematical population growth models.
- 3. Introduction to population genetic modelling.
- 4. Natural resources modelling.
- 5. Structured population modelling.
- 6. Demographic models.
- 7. Spatial dispersion models.
- 8. Ecosystem modelling (competition, predation, etc.)
- 9. Deterministic epidemic modelling.