



## Study Plan

**School:** School of Sciences and Technology

**Degree:** Bachelor

**Course:** Mathematics (cód. 734)

### 1st Year - 1st Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT12877L	Mathematical Analysis I	Mathematics	6	Semester	156
MAT00900L	Linear Algebra and Geometry I	Mathematics	6	Semester	156
MAT00932L	Discrete Mathematics	Mathematics	6	Semester	156
MAT14215L	Introduction to Logic and Foundations of Mathematics	Mathematics	6	Semester	156
MAT14234L	Geometry I	Mathematics	6	Semester	156

### 1st Year - 2nd Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT12878L	Mathematical Calculus II	Mathematics	6	Semester	156
MAT14237L	Introduction to Number Theory	Mathematics	6	Semester	156
MAT10689L	Mathematics and Statistics Laboratory	Mathematics	6	Semester	156
FIS13008L	General Physics I	Physics	6	Semester	156
INF11968L	Introduction to Programming	Informatics	6	Semester	156

### 2nd Year - 3rd Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT13046L	Mathematical Analysis III	Mathematics	6	Semester	156
MAT02354L	Probability and Statistics	Mathematics	6	Semester	156
MAT14239L	Algebra I	Mathematics	6	Semester	156
MAT14224L	Numerical Analysis I	Mathematics	6	Semester	156
MAT13638L	Operational Research	Mathematics	6	Semester	156

### 2nd Year - 4th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT13642L	Mathematical Analysis IV	Mathematics	6	Semester	156
MAT14226L	Topology	Mathematics	6	Semester	156
MAT14242L	Complements of Probability and Statistics	Mathematics	6	Semester	156



### 2nd Year - 4th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT10690L	Mathematics Programming	Mathematics	6	Semester	156
MAT00913L	Ordinary Differential Equations	Mathematics	6	Semester	156

### 3rd Year - 5th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT00903L	Complex Analysis	Mathematics	6	Semester	156
MAT14225L	Numerical Analysis II	Mathematics	6	Semester	156

### Group of Options of scientific area - MAT

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT00902L	Sampling	Mathematics	6	Semester	156
MAT13639L	Stochastic Processes	Mathematics	6	Semester	156
MAT13506L	Dynamic Optimization	Mathematics	6	Semester	156
MAT00904L	Functional Analysis	Mathematics	6	Semester	156
MAT14236L	Geometry II	Mathematics	6	Semester	156
MAT14241L	Computational Algebra	Mathematics	6	Semester	156
MAT13640L	Applied Statistics	Mathematics	6	Semester	156
MAT00919L	Multivariate Statistics	Mathematics	6	Semester	156
MAT00914L	Partial Differential Equations	Mathematics	6	Semester	156
MAT14223L	Differential Geometry	Mathematics	6	Semester	156
MAT14240L	Group theory	Mathematics	6	Semester	156
MAT14238L	Algebra II	Mathematics	6	Semester	156
MAT14235L	History of Mathematics	Mathematics	6	Semester	156



### 3rd Year - 5th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
<b>Group of Options of scientific area (MAT/INF/FIS/CEDU)</b>					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT00902L	Sampling	Mathematics	6	Semester	156
MAT13639L	Stochastic Processes	Mathematics	6	Semester	156
MAT13506L	Dynamic Optimization	Mathematics	6	Semester	156
MAT00904L	Functional Analysis	Mathematics	6	Semester	156
MAT14236L	Geometry II	Mathematics	6	Semester	156
MAT14241L	Computational Algebra	Mathematics	6	Semester	156
MAT13640L	Applied Statistics	Mathematics	6	Semester	156
MAT00919L	Multivariate Statistics	Mathematics	6	Semester	156
MAT00914L	Partial Differential Equations	Mathematics	6	Semester	156
MAT14223L	Differential Geometry	Mathematics	6	Semester	156
MAT14240L	Group theory	Mathematics	6	Semester	156
MAT14238L	Algebra II	Mathematics	6	Semester	156
MAT14235L	History of Mathematics	Mathematics	6	Semester	156
INF13203L	Machine Learning	Informatics	6	Semester	156
INF13194L	Programming II	Informatics	6	Semester	156
FIS13009L	General Physics II	Physics	6	Semester	156
PED02475L	Information and Communication Technologies in Education	Education Sciences	6	Semester	156
PED14141L	Foundations of Education	Education Sciences	6	Semester	156

### 3rd Year - 6th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT14220L	Measure and Integration	Mathematics	6	Semester	156
MAT14221L	Mathematics Seminar	Mathematics	6	Semester	156



### 3rd Year - 6th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
<b>Group of Options of scientific area - MAT</b>					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT00902L	Sampling	Mathematics	6	Semester	156
MAT13639L	Stochastic Processes	Mathematics	6	Semester	156
MAT13506L	Dynamic Optimization	Mathematics	6	Semester	156
MAT00904L	Functional Analysis	Mathematics	6	Semester	156
MAT14236L	Geometry II	Mathematics	6	Semester	156
MAT14241L	Computational Algebra	Mathematics	6	Semester	156
MAT13640L	Applied Statistics	Mathematics	6	Semester	156
MAT00919L	Multivariate Statistics	Mathematics	6	Semester	156
MAT00914L	Partial Differential Equations	Mathematics	6	Semester	156
MAT14223L	Differential Geometry	Mathematics	6	Semester	156
MAT14240L	Group theory	Mathematics	6	Semester	156
MAT14238L	Algebra II	Mathematics	6	Semester	156
MAT14235L	History of Mathematics	Mathematics	6	Semester	156



### 3rd Year - 6th Semester

Component code	Name	Scientific Area Field	ECTS	Duration	Hours
<b>Group of Options of scientific area (MAT/INF/FIS/CEDU)</b>					
Component code	Name	Scientific Area Field	ECTS	Duration	Hours
MAT00902L	Sampling	Mathematics	6	Semester	156
MAT13639L	Stochastic Processes	Mathematics	6	Semester	156
MAT13506L	Dynamic Optimization	Mathematics	6	Semester	156
MAT00904L	Functional Analysis	Mathematics	6	Semester	156
MAT14236L	Geometry II	Mathematics	6	Semester	156
MAT14241L	Computational Algebra	Mathematics	6	Semester	156
MAT13640L	Applied Statistics	Mathematics	6	Semester	156
MAT00919L	Multivariate Statistics	Mathematics	6	Semester	156
MAT00914L	Partial Differential Equations	Mathematics	6	Semester	156
MAT14223L	Differential Geometry	Mathematics	6	Semester	156
MAT14240L	Group theory	Mathematics	6	Semester	156
MAT14238L	Algebra II	Mathematics	6	Semester	156
MAT14235L	History of Mathematics	Mathematics	6	Semester	156
INF13203L	Machine Learning	Informatics	6	Semester	156
INF13194L	Programming II	Informatics	6	Semester	156
FIS13009L	General Physics II	Physics	6	Semester	156
PED02475L	Information and Communication Technologies in Education	Education Sciences	6	Semester	156
PED14141L	Foundations of Education	Education Sciences	6	Semester	156



## Conditions for obtaining the Degree:

\*\*\* TRANSLATE ME: Para obtenção do grau de licenciado em Matemática é necessário obter aprovação a 144 ECTS em unidades curriculares obrigatórias e 12 ECTS em unidades curriculares optativas, distribuídas da seguinte forma:

1<sup>o</sup> Ano

1<sup>o</sup> Semestre:

5 UC Obrigatórias num total de 30 ECTS

2<sup>o</sup> Semestre

5 UC Obrigatórias num total de 30 ECTS

2<sup>o</sup> Ano

3<sup>o</sup> Semestre

5 UC Obrigatórias num total de 30 ECTS

4<sup>o</sup> Semestre

5 UC Obrigatórias num total de 30 ECTS

3<sup>o</sup> Ano

5<sup>o</sup> Semestre

2 UC Obrigatórias num total de 12 ECTS

UC Optativas da área científica da Matemática oferecidas neste semestre num total de 12 ECTS

UC Optativas da área científica da Matemática/Informática/Física/Ciências da Educação oferecidas neste semestre num total de 6 ECTS

6<sup>o</sup> Semestre

2 UC Obrigatórias num total de 12 ECTS

UC Optativas da área científica da Matemática oferecidas neste semestre num total de 12 ECTS

UC Optativas da área científica da Matemática/Informática/Física/Ciências da Educação oferecidas neste semestre num total de 6 ECTS \*\*\*

## Program Contents

[Back](#)

### Mathematical Analysis I (MAT12877L)

1. Sequences and series.
2. Real functions of one variable.
3. Differential calculus.
4. Sequences and series of functions.
5. Integral calculus and applications.

[Back](#)

### Linear Algebra and Geometry I (MAT00900L)

Systems of linear equations.

Matrices.

Determinants.

Vector spaces.

Linear applications.

Eigenvalues and eigenvectors.

Geometry of plane and space.

Quadratic forms.



[Back](#)

### **Discrete Mathematics (MAT00932L)**

Sets

Induction

Combinatorics and counting

Recurrence

Graphs

Euclid algorithm

Modular arithmetic

[Back](#)

### **Introduction to Logic and Foundations of Mathematics (MAT14215L)**

Universe and language.

Axioms and set operations.

Relations and maps.

Equivalences and orders.

Well-ordered sets and ordinals.

Natural numbers and mathematical induction.

[Back](#)

### **Geometry I (MAT14234L)**

Euclidean geometry on the plane and on the three-dimensional space.

Criteria for congruence of triangles.

Ruler-and-compass constructions.

Ceva's theorem, Menelaus's theorem, Morley's trisector theorem.

Isometries. Similarities.

Symmetry. Symmetry groups.

Axiomatics. Finite geometries.

Circle inversion and the hyperbolic plane.

[Back](#)

### **Mathematical Calculus II (MAT12878L)**

1. Differential Calculus in  $\mathbb{R}^n$

Algebraic and topological structure of  $\mathbb{R}^n$ . Functions from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ : Continuity and the notion of limit. Differentiability.

Partial derivatives. Chain rule. Taylor's theorem in  $\mathbb{R}^n$  and applications to the study of extreme values. Inverse and implicit function theorems. Extreme values of functions with constrained variables

2. Integral Calculus in  $\mathbb{R}^n$

Multiple integrals: Fubini's theorem, change of variables theorem, applications to the computation of physical quantities. Line integrals: Integrals of scalar fields and vector fields. Fundamental theorem of calculus for line

integrals, conservative fields and scalar potentials. Green's theorem. Surface integrals: surface integrals of a scalar field, flux of a vector field, divergence theorem and Stokes' theorem.

[Back](#)

### **Introduction to Number Theory (MAT14237L)**

1. Divisibility

2. Congruences

3. Combinatorial Number Theory

4. Primitive Roots

5. Quadratic Residues

6. Arithmetic Functions

7. Continuous fractions



[Back](#)

### **Mathematics and Statistics Laboratory (MAT10689L)**

The programming in interactive system of symbolic and numerical calculation, and manipulation and visualization of data (mathematical packages SymPy, NumPy, Matplotlib and SciPy in Python, among others).

Introduction to the numerical methods of solving the nonlinear equations, data interpolation, numerical integration and differentiation, graphical visualization of the functions of one and two variables and optimization.

Introduction to Excel and R software. Elaboration of small functions in R.

Review of the basic concepts of statistics: population, sample and type of variables.

Univariate descriptive statistics: grouping of data, frequency table, graphical representation and summary statistics (location, dispersion, asymmetry, kurtosis and concentration). Empirical distribution function.

Bivariate descriptive statistics: graphical representation and contingency table.

[Back](#)

### **General Physics I (FIS13008L)**

#### I. Mechanics

- Scientific method. Measurements, units and dimensions.
- Kinematics and dynamics of mass points. Newton's laws and applications.
- Work and energy. Collisions and momentum. Conservation laws.
- Systems of many particles. The rigid body. Angular momentum.
- Universal gravitation.

#### II. Oscillations and waves

- Periodic and simple harmonic motion. Forced oscillations and resonance.
- Coupled oscillators. Normal modes.
- Progressive waves. The Doppler effect.
- Superposition and interference. Standing waves.

#### III. Option

##### A. Thermodynamics

- Thermal equilibrium and temperature.
- The ideal gas. The equation of state. Internal energy, heat and work.
- Calorimetry. Work and heat in thermal processes.
- The kinetic theory of gases.
- The 2nd law of thermodynamics. Heat engines. Reversible and irreversible processes. Entropy.

##### B. Topics on mechanical properties of solids and fluids.

- Stress, deformation, elasticity.
- Fluid mechanics.

[Back](#)

### **Introduction to Programming (INF11968L)**

Introduction to Programming with Python language

Use of interpreter in script and interactive modes

Variables, expressions and statements

Defining and Using Functions

Control structures

Native data structures

Sequential data structures: lists, tuples and strings.

Associative data structures: dictionaries.

Basics of input / output (I / O)

File manipulation

Graphic interface

Use of libraries

Libraries with advanced functionality for scientific calculation

Program development





[Back](#)

### **Mathematical Analysis III (MAT13046L)**

- 1 Introduction to Differential Geometry.
- 2 Introduction to Complex Analysis.
- 3 Ordinary Differential Equations.
- 4 Systems of ordinary differential equations.
- 5 Fourier series. Fourier integrals.

[Back](#)

### **Probability and Statistics (MAT02354L)**

- Probabilities and Conditional Probabilities.  
One and two-dimensional random variables (discrete and continuous).  
Moments. Moment and probability generating functions.  
Main probability distributions.  
Point estimation (moment and maximum likelihood estimation methods and properties of estimators).  
Confidence intervals for one and two populations.  
Hypothesis tests for one and two populations.  
Nonparametric alternatives for one and two populations.

[Back](#)

### **Algebra I (MAT14239L)**

- Groups  
Definition. Subgroups.  
Cosets. Lagrange's theorem.  
Normal subgroups. Quotient groups.  
Group homomorphisms. Cayley's theorem.  
Direct products.  
Abelian finite groups.  
Permutation groups.  
Matrices groups.

- Rings  
Definition.  
Subrings, ideals and quotient rings.  
Ring homomorphisms.  
Ring extensions.  
Polynomials.  
Matrices rings.  
Principal domains. Euclidean domains.  
Unique factorization domains.



[Back](#)

### **Numerical Analysis I (MAT14224L)**

- Floating point systems, errors, conditioning, convergence, stability.
- Nonlinear equations. Methods of bisection, Newton, fixed point iteration.
- Systems of linear equations. Direct methods: Gauss elimination, triangular factorizations, inverse matrix and determinant calculation. Matrix norms and condition numbers. Iterative methods: Jacobi, Gauss-Seidel, conjugate gradients.
- Systems of nonlinear equations.
- Interpolation and approximation of functions. Lagrange, Newton, Hermite polynomials, Chebyshev interpolation, splines, least squares approximation.
- Numerical derivation and integration. First and second order derivatives. Newton-Cotes quadrature rules, composite rules.
- Eigenvalues and eigenvectors. Localization and calculation.
- Implementation of some algorithms in an interactive numerical and symbolic calculation system.

[Back](#)

### **Operational Research (MAT13638L)**

- 1 - Introduction to Operations Research methodology
- 2 - Problem Formulation
- 3 - Linear Programming
- 4 - Duality; Shadow Prices and Opportunity Costs
- 5 - Optimization in networks and graphs.
- 6 - Project Management.

[Back](#)

### **Mathematical Analysis IV (MAT13642L)**

Difference equations. Non-linear difference equations.  
Integral transforms.  
Introduction to Partial Differential Equations: diffusion, Laplace, Wave, Black-Scholles.  
Introduction to measure theory.  
Introduction to convex analysis in  $\mathbb{R}^n$ .  
Applications.

[Back](#)

### **Topology (MAT14226L)**

Topological spaces. Open and closed sets. Cluster points and closure. Density. Hausdorff axiom. Sequences and convergence. Continuous functions. Compact spaces. Comparison of topologies. Subspaces and product of spaces. Tikhonov theorem. Complete metric spaces. Completion. Uniform continuity. Connected spaces. Banach and Hilbert spaces.

[Back](#)

### **Complements of Probability and Statistics (MAT14242L)**

- Complements of the Probability and Statistics theory: univariate probability distributions (Gamma, Beta and Weibull, among others) and multivariate, joint and conditional moments.
- Stochastic convergences, boundary distributions and some important inequalities.
- Transformations of variables and random vectors.
- Complements of hypothesis testing: more powerful test, Neyman-Pearson Lemma, likelihood ratio tests.
- Linear additive models: definitions, exponential family and its properties, link function. Simple and multiple linear regression model (estimation, adjustment, validation and forecast).



[Back](#)

### **Mathematics Programming (MAT10690L)**

Non-linear programming. Free optimization and optimization with constraints in the form of equality and inequality. Necessary and sufficient conditions of optimality, Lagrange multipliers, KKT conditions.

Numerical methods of optimization. Free optimization of functions of one and several variables. Constrained optimization: penalty functions, interior point method.

Integer and mixed programming. Multi-objective optimization. Heuristic algorithms.

Formulation of mathematical programming models using the modeling languages (AMPL, MathProg, GAMS, LINGO, etc).

Solving the models by software packages. Applications to Economics, Management, Natural Sciences and Engineering.

[Back](#)

### **Ordinary Differential Equations (MAT00913L)**

1. Ordinary Differential Equations. First order equations. Second order linear equations. Particular solution of non homogeneous equation. Homogeneous equation with constant coefficients.

2. Existence and Uniqueness of Solution. Inequalities and convergences. Picard's method of successive approximations. Solutions extension. Uniqueness theorems. Differential inequalities and extremal solutions. Continuous dependence of initial conditions.

3. Systems of Differential Equations. Existence and uniqueness of solutions. Linear systems. Systems with constant coefficients. Asymptotic behaviour of solutions

4. Stability of Solutions. Stability of quasi-linear systems. Planar autonomous systems. Limit cycles and periodic solutions. Lyapunov's method for autonomous and nonautonomous systems. Oscillatory equations

5. Boundary value problems. Green's functions. Maximum principle. Sturm-Liouville problems. Eigenfunction expansions. Nonlinear boundary value problems

[Back](#)

### **Complex Analysis (MAT00903L)**

Complex plane geometry and topology; holomorphic functions (Cauchy-Riemann equations, harmonic functions); elementary functions; complex integration (theorem of Cauchy, Cauchy's integral formula, Liouville's theorem, the fundamental theorem of algebra and the maximum modulus theorem); Taylor and Laurent series, analytic continuation; zeroes and poles, residue theorem (applications to integral calculus); Rouché's theorem; conformal transformations; further on harmonic maps.

[Back](#)

### **Numerical Analysis II (MAT14225L)**

1. Ordinary differential equations: initial value problems.

- Single step methods. Euler's methods. Truncation error and consistency. Taylor's methods. Runge-Kutta methods. Convergence, Stability.

- Multiple step methods. Adams, Nystrom and Milne methods. Multi-step linear methods. Consistency, convergence, stability. Predictor-corrector process.

- Systems of ordinary differential equations. Stiffness: a brief reference. Ordinary differential equations of higher order.

2. Ordinary differential equations: boundary-values problems.

- Collocation method. Least squares method.

- Finite difference method. Errors and convergence.

- Weak symmetrical formulation.

- Introduction to the finite element method. Basis functions.



[Back](#)

### **Sampling (MAT00902L)**

1. Basic notions on sampling and estimation.
2. Main steps about planning a sampling design and selection of sampling units.
3. Methods for data collection in survey sampling.
4. Simple random sampling.
5. Estimation of totals, means, proportions and ratios.
6. Ratio and regression estimation.
7. Stratified sampling.
8. Cluster and multi-step sampling designs.
9. Unequal probability sampling.

[Back](#)

### **Stochastic Processes (MAT13639L)**

1. General concepts of stochastic processes:
  - Properties and classification;
2. Discrete-time Markov chain
  - Transition probability matrices;
  - Chapman-Kolmogorov equations;
  - Classification of states;
  - Limiting theorems;
  - Simple branching processes;
3. Poisson processes:
  - Axiomatic;
  - Waiting times in Poisson processes;
4. Continuous-time Markov chain.
  - Birth-Death processes
  - Introduction to queueing theory

[Back](#)

### **Dynamic Optimization (MAT13506L)**

Optimization of dynamic systems and processes, control problems. Historical introduction.

Calculation of variations. Important particular examples: geodetic, brachistochrone problem, revolution surfaces of minimal area. Euler's equation. Condition of transversality.

Control theory. Controllability. Optimal control problems. Examples in Economics and Management. Pontryagin's maximum principle. Model of optimal economic growth. Control problem in discrete time.

Dynamic programming. Multistage decision processes. Bellman's principle of optimality. Typical problems of dynamic programming.

[Back](#)

### **Functional Analysis (MAT00904L)**

Finite and infinite dimensional Banach spaces.

Hahn-Banach theorem.

Strong and weak convergence.

Hilbert spaces.

Bounded linear operators. Compact operators. Fixed point theory.

Integral equations. Fredholm alternative.

Resolvent and spectrum.



[Back](#)

### **Geometry II (MAT14236L)**

From perspective to projective geometry.  
Projective plane and projective space.  
Desargues theorem and Pappus theorem.  
Projectivities.

[Back](#)

### **Computational Algebra (MAT14241L)**

Polynomial rings and ideals.  
Monomial orderings in  $k[x_1, \dots, x_n]$ .  
Monomial ideals and Dickson lemma.  
Hilbert basis theorem and Gröbner bases.  
Buchberger's algorithm.  
Some applications of Gröbner bases.  
Rewriting systems. Knuth Bendix algorithm.

[Back](#)

### **Applied Statistics (MAT13640L)**

- Analysis of variance models: fixed effects and random effects (single and multiple factor). Multiple comparisons. Other approaches when assumptions are not verified.
- Analysis of Covariance.
- Introduction to Categorical Data Analysis. Contingency Tables. Correlation measures with at least one categorical variable. Characterization of a generalized linear model. Models with categorical response variables.
- Introduction to Principal Component Analysis.
- Introduction to Cluster Analysis.

[Back](#)

### **Multivariate Statistics (MAT00919L)**

Exploratory Analysis of Multivariate Data  
Correspondence Analysis  
Multidimensional Scaling  
Decision Trees  
Software: SPSS and R

[Back](#)

### **Partial Differential Equations (MAT00914L)**

Linear and nonlinear equations.  
Hyperbolic, parabolic and elliptic equations: "classification and canonical forms".  
III- and well-posed problems.  
The problems for the wave, the diffusion and the Laplace equations.  
Separation of variables, Fourier series (a motivation to the Hilbert spaces).  
Introduction to the Fourier transform (a motivation to the Sobolev spaces).  
Distributions, weak solutions and Green functions.  
Characteristics and evolution (a motivation to the energy and variational methods; implications for the nonlinear equations).



[Back](#)

### **Differential Geometry (MAT14223L)**

Review of topological spaces and metric spaces; completeness. Calculus on normed vector spaces. Notions of manifold, tangent space, tangent vector field, morphism between manifolds. Study of submanifolds; theorems on the construction of manifolds by inverse image; applications to Lie groups; introduction to the study of Riemannian manifolds; homogeneous manifolds.

[Back](#)

### **Group theory (MAT14240L)**

Free Abelian groups.

Finitely generated Abelian groups.

The action of a group on a set.  $p$ -groups.

Sylow theorem.

Nilpotent and soluble groups.

Free groups. Generators and relations.

Dehn problems: word problem, conjugation problem, and isomorphism problem.

[Back](#)

### **Algebra II (MAT14238L)**

Divisibility criteria. Field extensions.

Applications of Galois theory to classical problems.

Ruler and compass constructions.

Finite fields.

[Back](#)

### **History of Mathematics (MAT14235L)**

1. Aspects of Babylonian, Egyptian, and Greek mathematics in ancient history. Conceptions of infinity of ancient philosophers. Pythagoreans, Eudoxus and the question of incommensurability. Euclid and the axiomatic method. The "method" of Archimedes. Zeno and the motion paradoxes. Plato and Platonism. Aristotle and logic.

2. Medieval mathematics and Renaissance. Fibonacci. Italian algebraists. Pedro Nunes. Cavalieri, Torricelli, Pascal, Kepler, and Galileo's indivisibles.

3. Descartes, Fermat, Newton, and Leibniz. Birth of analytic geometry and infinitesimal calculus. The importance of series.

4. Important aspects of mathematics from 17th to 20th century: conceptions of mathematical infinity, analysis' rigour, algebraic solvability, non-Euclidean geometries, impossibility theorems, set theory and other questions of foundations of mathematics.

5. Most important philosophical trends and their representatives: Platonism, logicism, formalism, intuitionism, empiricism, nominalism.

[Back](#)

### **Machine Learning (INF13203L)**

Basic concepts

Machine Learning paradigms: supervised, unsupervised, reinforcement learning

Supervised learning: classification and regression

Binary, multi-class and multi-label classification

Algorithms: logistic regression, perceptron, decision trees, rules, naive Bayes, support vector machines

ML practice: overfitting, bias/variance tradeoff, model selection (train/test, holdout, cross-validation), confusion matrix and evaluation metrics (accuracy, error, precision, recall, others)

Unsupervised learning: clustering

Algorithms: K-means, EM

Clustering evaluation measures

Introduction to ensemble methods



[Back](#)

### **Programming II (INF13194L)**

Object-oriented analysis and program design.

A general-purpose object-oriented language (Java).

Incremental program development.

Class libraries (packages).

Simple graphical user interfaces.

[Back](#)

### **General Physics II (FIS13009L)**

I. Electromagnetism

Electrostatics. Electric charges and forces. Gauss's law

Electric potential. Capacitors

Electric current. Kirchhoff's rules. RC circuits

Magnetic fields and the Lorentz force

Sources of the magnetic field. Magnetism in matter

Electromagnetic induction. Faraday's law

AC-current

Maxwell's equations

Electromagnetic waves. Polarization

II. Optics

Nature of light. Geometric optics. Image formation by mirrors and lenses

Wave optics. Double-slit experiment. Diffraction and interference

III. Modern physics

Special relativity. Time dilation and Lorentz contraction. Relativistic momentum and energy

Introduction to quantum physics. Particle properties of light. Photoelectric effect and Compton scattering. Waveparticle duality. Uncertainty principle. Wave function

Atoms. Atomic spectra. Hydrogen atom in quantum mechanics. Periodic table of the elements

Nuclear physics. Stability and instability of nuclei. Elementary particles. Contemporary physics.

[Back](#)

### **Information and Communication Technologies in Education (PED02475L)**

1. Rationale for the use of Technology in Education

a. Reasons for using technology in education

b. Learning and Technology

c. Digital literacy: concepts and models

2. Design and planning programs and projects in the field of digital literacy

a. Curricular structure of programs and projects

b. Selection and evaluation of technologies and digital resources

c. Planning tools: teaching guides and lesson plans

d. Methodologies of educational work: ICT and project work, and methods of collaborative work.

e. Tools, resources and environments to support the assessment of student learning (digital portfolios).

f. Safety, ethics and protection of children and young people in using ICT

g. Social networks and community learning: the new social challenges

3. Evaluation of digital educational resources

a. Types of software and educational resources.

b. Selection and evaluation of digital educational resources, concepts, tools and criteria

4. Special educational needs and ICT



[Back](#)

### **Foundations of Education (PED14141L)**

Education as a symbolic construction of the human

#### 2. The Foundations of Education

2.1. Biological foundations

2.2. Psychological foundations

2.3. Anthropological foundations

2.4. Socio-economic foundations

2.5. Philosophical foundations

3. Education and worldviews

4. Education and political agendas; national and international

5. Education as a profession: nature, missions and challenges

[Back](#)

### **Measure and Integration (MAT14220L)**

Jordan and Lebesgue measure in a finite dimensional space. Measurable sets. Abstract measure. Extension and completion. Measurable functions. Lebesgue integral. Convergence almost everywhere and by measure. Convergence theorems. Product of measures. Fubini theorem. Sign measures. Radon-Nikodym theorem. Differential inequalities and extremal solutions. Continuous dependence of initial conditions. Applications.

[Back](#)

### **Mathematics Seminar (MAT14221L)**

Seminars will address contents associated to themes on which speakers develop their present or past scientific research, supported by their scientific publications.